McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-466A

COMPLEX ANALYSIS

Examiner: Professor I. Klemes
Associate Examiner: Professor H. Darmon

Date: Friday, December 5, 1997
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Show all work.
Answer all 8 questions.
Each question is worth 10 marks.

This exam comprises the cover and 1 page of questions.

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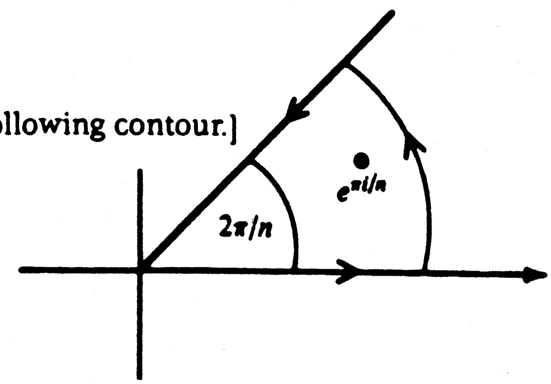
1. Show that if f is analytic in $|z| \leq 1$, there must be some positive integer n such that $f(1/n) \neq 1/(n+1)$.

2. Suppose f is bounded and analytic in $\operatorname{Im} z \geq 0$ and real on the real axis. Prove that f is constant.

3. Evaluate

$$\int_0^\infty \frac{dx}{1+x^n}$$

where $n \geq 2$ is a positive integer. [*Hint*: Consider the following contour.]



4. Classify the singularities of

a. $\dfrac{1}{z^4+z^2}$

b. $\cot z$

5. Does there exist a function f with an isolated singularity at 0 and such that $|f(z)| \sim \exp(1/|z|)$ near $z = 0$?

6. Find a conformal mapping f between the regions S and T, where
$S = \{z = x + iy : -2 < x < 1\}$; $T = D(0; 1)$.

7. Find the number of zeroes of
$f(z) = z^6 - 5z^4 + 3z^2 - 1$ in $|z| \leq 1$.

8. Suppose f is analytic inside and on a regular closed curve γ and has no zeroes on γ. Show that if m is a positive integer then

$$\frac{1}{2\pi i}\int_\gamma z^m \frac{f'(z)}{f(z)}\,dz = \sum_k (z_k)^m$$

where the sum is taken over all the zeroes of f inside γ. (with multiplicity)