1. Consider the system of inequalities:

$$\begin{aligned} x_1+2x_2+\ x_3+x_4 &\le 10 \\ x_1-\ x_2-2x_3+x_4 &\le\ 3 \end{aligned}$$

$$x_i \ge 0,\ \ i=1,2,3,4.$$

Its solutions determine a set F.

(a) Find all extreme points of F.

(b) Is the point $x = (5\ 2\ \frac{1}{3}\ \frac{2}{3})^T$ an extreme point of F?

2. (a) Find all values of the parameter α for which the feasible set of the problem

$$\begin{aligned} &\text{Max } x_1+\ x_2+\ x_3 \\ &\text{s.t.} \\ &\qquad x_1+\alpha x_2-\ x_3 \le 10 \\ &\qquad x_1+4x_2+\alpha x_3 \le 2 \end{aligned}$$

$$x_i \ge 0,\ \ i=1,2,3$$

is unbounded.

(b) Fix $\alpha = 1$ and solve the above linear program. Then, if $b_1 = 10$ is increased by $\Delta b_1 = 0.1$, if $b_2 = 2$ is increased by $\Delta b_2 = 0.2$, and if $a_{21} = 1$ is decreased by $\Delta a_{21} = -0.2$, estimate the corresponding change of the optimal value function. Use shadow prices.

3. Solve the linear program

$$\begin{aligned} &\text{Max } \ x_1+x_2+\ x_3+2x_4 \\ &\text{s.t.} \\ &\qquad x_1+x_2+2x_3+\ x_4 = 4 \\ &\qquad 2x_1-x_2+\ x_3-\ x_4 \ge 1 \end{aligned}$$

$$x_i \le 0,\ \ i=1,2,3,4$$

by the simplex method.

4. Using the Karush-Kuhn-Tucker conditions check whether

$$x_1^* = \frac{5}{3},\ x_2^* = 0,\ x_3^* = 0,\ x_4^* = \frac{7}{3}$$

is an optimal solution of the program from Problem 3.

5. Consider the problem

$$\begin{array}{l} \text{Opt } x_1^2 + 2x_1 - x_2^2 \\ \text{s.t.} \\ \qquad x_1^2 + \qquad x_2^2 + x_3^2 = 1. \end{array}$$

Using the first and second order optimality condition of your choice, determine whether

$$x_1^* = -\frac{1}{2},\ x_2^* = \frac{\sqrt{3}}{2},\ x_3^* = 0$$

is a local optimum.

6. Joe has \$10,000 to invest in 3 mutual funds: A, B and C. After studying their performance over the last 5 years, Joe has calculated the covariance matrix to be

$$C = \begin{bmatrix} 12 & -5.6 & 23 \\ -5.6 & 2.8 & -12 \\ 23 & -12 & 55.2 \end{bmatrix}.$$

The expected future average returns from the three investments are, respectively,

$$E_1 = 9,\ E_2 = 7,\ E_3 = 10$$

cents per dollar per year.
Joe has two requirements: (1) The combined expected yearly return from his investment must be no less than \$800 and (2) the variance in future (yearly) dividend payments should be as small as possible. How much should Joe invest in each fund to achieve these requirements? You are asked to:

(a) Formulate this portfolio problem as a convex program.

(b) Check whether the investment of \$5,000 in each of the funds A and B, and \$0 in C is optimal.

Is the optimal value more sensitive to small changes in the investment (\$10,000) than in the lower bound (\$800)?

7. Consider 4 decision making units A, B, C and D, each with 2 inputs and 2 outputs given in the table below:

	Inputs		Outputs	
A	2	3	1	2
B	1	2	1	1
C	3	4	2	6
D	1	3	1	2

After applying the Charnes-Cooper-Rhodes efficiency test to the decision making unit A, it has been found that an optimal solution is

$$x_1^* = 0,\ x_2^* = \frac{1}{3},\ y_1^* = \frac{2}{3},\ y_2^* = 0.$$

Also, that the corresponding shadow prices (for the inequality constraints) are

$$p_1^* = 0,\ p_2^* = \frac{1}{2},\ p_3^* = \frac{1}{4},\ p_4^* = 0\ .$$

Using this information, determine whether A is efficient. If it is not, find its reference set and its Charnes- Cooper-Rhodes projection ("ideal" DMU for A) on the efficiency frontier.

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-417A/487A

MATHEMATICAL PROGRAMMING

Examiner: Professor S. Zlobec | Date: Wednesday, December 18, 1996
Associate Examiner: Professor G. Schmidt | Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Attempt all problems
Explain the answers
Calculators are Permitted

This exam comprises the cover and 3 pages of questions.