1. Given that $u(x, y) = x^3 + kxy^2$ is a harmonic function, find the value of the constant k. Also, find a function $v(x, y)$ such that $u + iv$ is analytic.

2. (a) Simplify $(-\sqrt{3} - i)^{-5}$.

   (b) If a branch of $z^{1/3} = h(z)$ satisfies

$$h(1) = -\frac{1}{2} + i\frac{\sqrt{3}}{2},$$

   and has branch cut $x \le 0,\ y = 0$, evaluate $j(i)$.

3. Evaluate:

   (a) $\displaystyle\int z^3 \cos(1/z)dz$ around $|z + 1 + i| = 4$;

   (b) $\displaystyle\int_0^\pi \frac{\cos\theta}{5 + 4\cos\theta}d\theta$.

4. Showing all main steps in the contour integration method, evaluate

$$\int_0^\infty \frac{x\sin 3x}{(x^2 + 4)^2}dx \ .$$

5. Find the Inverse Fourier Transform of the function

$$F(\omega) = \frac{e^{-2i\omega}}{9\omega^2 + 1} \ , \quad -\infty < \omega < \infty \ .$$

6. For the function

$$F(s) = \frac{se^{-5s}}{(s + 1)^3} \ ,$$

   (a) set up the Bromwitch integral for the inverse Laplace transform $f(t)$, including a diagram of the path and all relevant details concerning $F(s)$.

   (b) Evaluate the integral in (a), explaining the steps for all cases of t, $-\infty < t < \infty$.

7. (a) In the annulus $2 < |z| < 3$, find the Laurent series of

$$f(z) = \frac{z^3 + 1}{z^2 + z - 6} \ .$$

   (b) Find the $\mathbb{Z}$ transform of $f(t) = e^{2it}$, and find a closed form expression for your series.

   (c) Find the inverse $\mathbb{Z}$ transform of $e^{1/z}$.

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-381B

COMPLEX VARIABLES & TRANSFORMS

Examiner: Professor I. Klemes   Date: Tuesday, April 29, 1997
Associate Examiner: Professor D. Sussman   Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Answer all 7 questions
Simplify your answers
Calculators Not Permitted

This exam comprises the cover and 1 page of questions.