McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 355

Analysis 4

Examiner: Professor J. Toth
Associate Examiner: Professor D. Jakobson

Date: April 29, 2004
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Calculators are not permitted.
Each question is worth 10 points.
Please show all your work.

This exam comprises the cover, and 1 page of 5 questions.

Mathematics 355 : Final Examination

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1. Assume that $f(x,y)$ and $\frac{\partial}{\partial x}f(x,y)$ are both continous functions on $R \times R$ and let $[a,b] \subset R$ be any finite interval. Show that,

$$\frac{\partial}{\partial x}\int_a^b f(x,y)dy = \int_a^b \frac{\partial}{\partial x}f(x,y)\,dy.$$

Justify each step your argument carefully.

2. Let $a > 0$ and $f(x)$ be a continuous function on $[-a,a]$. Compute the limit

$$\lim_{N\to\infty} N^{1/2}\int_{-a}^{a} e^{-Nx^2/2}f(x)dx.$$

Justify each step of your argument carefully. (Hint: Make a change of variables in the integral).

3. Let $(X,\mathcal{F},\mu)$ be a measure space, A and B be measurable subsets of X and $S(A,B) := (A-B)\cup(B-A)$. Show that, if $\mu(S(A,B)) = 0$, then, for every nonnegative measurable function f,

$$\int_A f d\mu = \int_B f d\mu.$$

4. Let $B^n := \{x \in R^n; |x| \le 1\}$ be the unit ball in Cartesian n-space with $v_n :=$ Volume (B^n). Show that

$$v_n = 2v_{n-1}\cdot\int_0^1 (1-t^2)^{\frac{n-1}{2}}\,dt.$$

Justify each step of your argument carefully. (Hint: Use Fubini's theorem.)

5. Let $\chi_{[-1,1]}$ be the indicator function of the interval $[-1,1]$ (ie. its characteristic function).
(i) Compute $\widehat{\chi_{[-1,1]}}(y)$.
(ii) Use the Plancherel formula to compute

$$\int_{-\infty}^{\infty}\left(\frac{\sin x}{x}\right)^2\,dx.$$