1. (a) Let S be a rotation about a point A through an angle α and P a rotation about B through an angle β. Describe the isometry given by the formula $R = PSP^{-1}$.

   (b) A finite figure is a figure whose symmetry group does not contain nontrivial translations. Show that a finite figure cannot have two rotation symmetries with different rotocenters.

2. Give the definition of isomorphic groups. Is C_6 isomorphic to D_3? Justify your answer.

3. Identify the following isometries:

   (a) $(x_1, x_2) \to (-x_1 + 3, -x_2 + 5)$

   (b) $(x_1, x_2) \to (x_1 + 6, -x_2 - 1)$.

4. Find analytic formulas for the following isometry: $M_1 M_2$, where M_1 is the reflection in $x_1 - x_2 + 1 = 0$, and M_2 is the reflection in $-x_1 + x_2 = 0$,

5. Prove that if $\{u, v, w\}$ is an orthonormal triple, then for all $x \in R^3$

$$x = \langle x, u\rangle u + \langle x, v\rangle v + \langle x, w\rangle w.$$

6. Explain how to measure angles on S^2. Find the angles of the triangle PQR (or their cosines), where $P = (0, -1/\sqrt{2}, -1/\sqrt{2})$, $Q = (0, 1/\sqrt{2}, -1/\sqrt{2})$, $R = (1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$.

7. Define the projective plane P^2 and the mapping $T : E^2 \to P^2 - l_\infty$. What are the properties of this mapping?

8. (a) Define intersecting, parallel, and ultraparallel lines of H^2. How to find the point of intersection of two intersecting lines? If $\zeta = (1, 1, 1)$ and $\nu = 1/\sqrt{3}(2, 0, 1)$, what can you say about the lines l and m with respective unit normals ζ and ν?

   (b) Find all lines through $P = (2, 0, \sqrt{5})$ which are ultraparallel to the line with the normal vector $\zeta = (1, 1, 1)$.

9. What is "special relativity" . What is length contraction? What is time dilation? Deduce these phenomena from the Lorenz transformation formulas.

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-348A

TOPICS IN GEOMETRY

Examiner: Professor O. Kharlampovich
Associate Examiner: Professor A. Volkov

Date: Monday, December 13, 1999
Time: 9:00 A.M. - 12:00 Noon.

INSTRUCTIONS

Calculators are permitted.

This exam comprises the cover and one page of questions.