Problems $1, 2, 3, 4, 5, 6, 7$ from the section on Isometry and Symmetry

1. Determine the isometry $\mathcal{S}_{90^\circ}\mathcal{R}$.

2. Determine the isometry $\mathcal{G}\mathcal{C}_{60^\circ}$, where $\mathcal{G}$ is the glide reflection represented by the half-arrow $\overline{BA}$, and $\mathcal{C}_{60^\circ}$ is the rotation with center C and angle counterclockwise 60°. (The triangle ABC is equilateral.)

3. The pool table in the figure below has pockets only at the corners. Into which pocket does the ball go? Justify your answer.

4. In triangle ABC "inscribe" a line segment (the ends of the constucted segment should be on the sides of the triangle) equal and parallel to the given segement a.

5. Draw a frieze (strip) pattern whose group of symmetries is generated by the half turn $\mathcal{H}$ (with center indicated below) and the reflection $\mathcal{R}$ (in the line indicated below). Indicate with an appropriate symbol the isometry $\mathcal{HRHRH}$.

6. Pair each frieze in the left column with a frieze in the right column if they have the same group of symmetries. Describe the symmetry group of each of the seven strips by indicating a set of generators for the strips in the left column.

7. Indicate (draw) for each of the six ornamental patterns a set of generators of its symmetry group. Have you recently seen the brick pattern at the bottom on the right? Where?

Problem 8 on Equidecomposability

8. (i) State briefly the Bolyai-Gerwin theorem.

   (ii) State briefly the Hadwiger-Glur theorem.

   (iii) Explain why it it is impossible to dissect a triangle into a finite number of pieces and move the pieces using translations only to make a parallelogram.

Problem 9. On a geometric inequality

9. Let $ABCD$ be a convex quadrilateral (diagram below). Prove that

$$\overline{AB} + \overline{CD} < \overline{AC} + \overline{BD}$$

($\overline{PQ}$ denotes the length of segment PQ.)

Problem 10 from the section of Problems in Discrete Geomtry

10. The answer to a certain question is:

    NO, as shown by the sequence

    $$\mathbf{n, n-1, \ldots, 1, 2n, 2n-1, \ldots, n+1, \ldots, n^2, n^2-1, n^2-2, \ldots, n^2-n+1.}$$

    What was the question?

Problem 11: On Hyperbolic geometry

11. (i) State the Hyperbolic Axiom of Parallelism.

    (ii) Explain the distiction between the statements (a) and (b):

    (a) the lines k and m are parallel;

    (b) the line k and m are ultra-parallel.

    (iii) Assuming the construction for the common perpedicular to two ultra-parallel lines, describe the construction for the common parallel to two non-parallel rays

    (iv) Given two ultra-parallel lines ℓ and m, how many lines are parallel to both ℓ and m. (Supply a suitable diagram.)

McGILL UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAMINATION

MATHEMATICS 189-348A

TOPICS IN GEOMETRY

Examiner: Professor W. O. J. Moser — Date: Thursday, Dec. 12, 1996

Associate Examiner: Professor W. Brown — Time: 9:00–12:00 hrs

Instructions

You MUST return this booklet.

You may write all or some or none of your solutions in this booklet.

You may write all or some or none of your solutions — or your rough work — in a "normal" examination booklet.

Print your name______________________________

Sign your name______________________________

Print your student number ______________________________