MARKS

1. Suppose that for independent identically distributed random variables $X_1, \ldots, X_n$, $\mathbb{E}(X_i) = \mu$ and $\operatorname{Var}(X_i) = \sigma^2$.

(4) (a) Show that $\operatorname{Var}(\overline{X}) = \dfrac{\sigma^2}{n}$.

(4) (b) Find $\mathbb{E}\left\{\sum^n X_i^2 - n\overline{X}^2\right\}$.

(4) 2. (a) If T is an estimator of $\tau(\theta)$ show that Mean Square Error of T is $\operatorname{Var}(T) + [b(T)]^2$ where $b(T) = \mathbb{E}(T) - \tau(\theta)$, the bias.

(8) (b) If $X_1, \ldots, X_n$ are independent $U[0, \theta]$ compare the mean square errors of $2\overline{X}$, the moment estimator of θ and the maximum likelihood estimator of θ.

(4) 3. (a) Suppose that X_i $(i = 1, \ldots, n)$ are independent Bernoulli random variables, $\operatorname{Ber}(1, p)$. Find a uniformly minimum variance unbiased estimator of $p(1-p)$.

(8) (b) If X_i, $i = 1, 2, \ldots, n$, are independent $N(\alpha + \beta y_i, \sigma^2)$ find the maximum likelihood estimates of α, β and σ^2. ($y_1, \ldots, y_n$ are known constants.)

(4) 4. (a) In problem 3(b) what are the least-squares estimates of α and β?

(9) (b) Under the assumption of normality as in 3(b) state the distributions of

(1) $\hat{\alpha}$, $\hat{\beta}$, (2) $\dfrac{n\hat{\sigma}^2}{\sigma^2}$, where $\hat{\alpha}, \hat{\beta}, \hat{\sigma}^2$ are estimates of α, β and σ^2.

(7) (c) Explain briefly how one can construct a 90% confidence interval for α (σ^2 unknown).

(9) 5. (a) State and prove the Neyman-Pearson lemma.

(6) (b) What is a uniformly most powerful test? Derive the uniformly most powerful test of $H_0 : \mu = 2$ against $\mu < 2$ when $X_1, \ldots, X_n$ is a random sample from $N(\mu, 1)$. Determine the power $\pi(1.75)$ when it is known that $\overline{X} = 2.35$ and $n = 9$.

(10) 6. (a) Independent random samples $X_i \sim N(\mu_1, \sigma_1^2)(i = 1, \ldots, n)$ and $Y_j \sim N(\mu_2, \sigma_2^2)$ $(j = 1, \ldots, m)$ are being tested to see if $H_0 : \mu_1 = \mu_2$ against $H_A : \mu_1 \neq \mu_2$ at $\alpha = 0.10$ level of significance. Given that $n = m = 9$, $\overline{X} = 16$, $\overline{Y} = 10$, $s_1^2 = 36$ and $s_2^2 = 45$ perform the test and state your conclusion.

(10) b) If the data were treated as a paired sample (X_i, Y_i) and s_D^2, the sample variance of the difference $X_i - Y_i$ perform an appropriate test of $\mu_1 = \mu_2$ against the alternative that $\mu_1 \neq \mu_2$ at $\alpha = 0.10$ level.

(5) 7. (a) In a certain genetic experiment it is believed that brown will occur with probability p_b, white with probability p_w and spotted with probability p_s. Suppose that $p_w = p_b = \frac{1}{2}p_s$ and in 40 trials the following results are observed.

	Brown	White	Spotted
observed	5	15	20

Test the hypothesis that the model is appropriate at a 10% level of significance

(8) (b) A sample of 750 people was selected and classified according to income and stature with the following results.

Income

Stature	Poor	Average	Rich
Thin	120	60	50
Average	50	200	70
Fat	100	50	50

Test the hypothesis that Income and Stature are independent at $\alpha = 0.10$.

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-324A

INTRODUCION TO STATISTICS

Examiner: Professor V. Seshadri | Date: Friday, December 6, 1996
Associate Examiner: Professor K. Worsley | Time: 9:00 A.M. - 12:00 Noon

INSTRUCTIONS

Answer ALL the questions
Simple calculators are permitted

This exam comprises the cover, 2 pages of questions and four pages of tables.