STUDENT NAME: STUDENT ID#

McGILL UNIVERSITY
FACULTY OF SCIENCE

FINAL EXAMINATION

MATH323B

PROBABILITY THEORY

Examiner: Professor M. Asgharian
Associate Examiner: Professor Anderson

Date: Wednesday, April 12, 2006
Time: 2:00 PM - 5:00 PM .

INSTRUCTIONS

Answer ONLY 7 questions.
Calculators are permitted.
Answer directly on exam
Dictionaries are allowed

Questions	Marks
1	
2	
3	
4	
5	
6	
7	
8	

This exam comprises the cover, 8 pages of questions,and page 9 which is blank, and 2 pages of formulas.

1. The eight-member Human Relations Advisory Board of Montreal considered the complaint of a woman who claimed discrimination based on sex, on the part of a local company. The board, composed of five women and three men, voted 5-3 in favor of the plaintiff, the five women voting in favor of the plaintiff, the three men against. The attorney representing the company appealed the board's decision by claiming sex bias on the part of the board members. If there was no sex bias among the board members, it might be reasonable to conjecture that any group of five board members would be as likely to vote for the complaint as any other group of five. If this were the case, what is the probability that the vote would split along sex lines(five women for, three men against)? (*10 marks*)

2. A bowl contains w white balls and b black balls. One ball is selected at random from the bowl, its color is noted, and it is returned to the bowl along with n additional balls of the same color. Another single ball is randomly selected from the bowl (now containing w+b+n balls) and it is observed that the ball is black. What is the (conditional) probability that the first ball selected was white? (*10 marks*)

3. The number of cars driving past a parking area in a 1-minute time interval has a Poisson distribution with mean λ. The probability that any individual driver actually wants to park his or her car is p. Assume that individuals decide whether to park independently of one another.

   (a) If one parking place is available and it will take you 1 minute to reach the parking area, what is the probability that a space will still be available when you reach the lot? (Assume that no one leaves the lot during the 1-minute interval.) (*5 marks*)

   (b) Let W denote the number of drivers who wish to park during 1-minute interval. Derive the probability distribution of W. (*5 marks*)

4. Suppose that plants of a particular species are randomly dispersed over an area, so that the number of plants in a given area follows a Poisson distribution with a mean density of λ plants per unit area. If a plant is randomly selected in this area, find the probability density function of the distance to the *nearest* neighboring plant. [*Hint:* If R denotes the distance to the nearest neighbor, then $P(R > r)$ is the same as the probability of seeing no plants in a circle of radius r.]

5. A retail grocer has a daily demand Y for a certain food sold by the pound, where Y (measured in hundreds of pounds) has a probability density function given by

$$f(y) = \begin{cases} 3y^2, & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(She cannot stock over 100 lbs.) The grocer wants to order 100k pounds of food. She buys the food at 5 cent per pound and sells it at 10 cent per pound. What value of k will maximize her expected daily profit? *(10 marks)*

6. In a clinical study of a new drug formulated to reduce the effects of rheumatoid arthritis researchers found that the proportion p of patients who respond favorably to the drug is a random variable that varies from batch to batch of the drug. Assume that p has a probability density function given by

$$f(p) = \begin{cases} 12p^2(1-p), & 0 \leq p \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Suppose that n patients are injected with portions of the drug taken from the same batch. Let Y denote the number showing a favorable response.

(a) Find the unconditional probability distribution of Y for general n. (*5 marks*)

(b) Find $E(Y)$ for $n = 2$. (*5 marks*)

7. In a missile-testing program, one random variable of interest is the distance between the point at which the missile lands and the center of the target at which the missile was aimed. If we think of the center of the target as the origin of a coordinate system, we can let Y_1 denote the north-south distance between the landing point and the target center and let Y_2 denote the corresponding east-west distance. (Assume that north and east define positive directions.) The distance between the landing point and the target center is then $U = \sqrt{Y_1^2 + Y_2^2}$. If Y_1 and Y_2 are independent, standard normal random variables, find the probability density function for U. *(10 marks)*

8. Suppose the probability that a person will suffer an adverse reaction from a medication is .001. What is the probability that 2 or more will suffer an adverse reaction if the medication is administered. [*Hint:* Use Poisson approximation.] (*10 marks*)

Discrete distributions

Distribution	Probability Function	Mean	Variance	Moment-Generating Function
Binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y};$ $y = 0, 1, \ldots, n$	np	$np(1-p)$	$[pe^t + (1-p)]^n$
Geometric	$p(y) = p(1-p)^{y-1};$ $y = 1, 2, \ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Hypergeometric	$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}};$ $y = 0, 1, \ldots, n$ if $n \le r$, $y = 0, 1, \ldots, r$ if $n > r$	$\frac{nr}{N}$	$n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$	
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!};$ $y = 0, 1, 2, \ldots$	λ	λ	$\exp[\lambda(e^t - 1)]$
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r};$ $y = r, r+1, \ldots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]^r$

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Continuous distributions

Distribution	Probability Function	Mean	Variance	Moment-Generating Function
Uniform	$f(y) = \frac{1}{\theta_2 - \theta_1}; \theta_1 \le y \le \theta_2$	$\frac{\theta_1 + \theta_2}{2}$	$\frac{(\theta_2 - \theta_1)^2}{12}$	$\frac{e^{t\theta_2} - e^{t\theta_1}}{t(\theta_2 - \theta_1)}$
Normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2\right]$ $-\infty < y < +\infty$	μ	σ^2	$\exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$
Exponential	$f(y) = \frac{1}{\beta}e^{-y/\beta}; \quad \beta > 0$ $0 < y < \infty$	β	β^2	$(1 - \beta t)^{-1}$
Gamma	$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta};$ $0 < y < \infty$	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta t)^{-\alpha}$
Chi-square	$f(y) = \frac{(y)^{(v/2)-1} e^{-y/2}}{2^{v/2}\Gamma(v/2)};$ $y^2 > 0$	v	$2v$	$(1 - 2t)^{-v/2}$
Beta	$f(y) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y^{\alpha-1}(1-y)^{\beta-1};$ $0 < y < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	does not exist in closed form