McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 323

Probability

Examiner: Professor W. Anderson
Associate Examiner: Professor M. Asgharian

Date: Tuesday December 20, 2005
Time: 9:00 a.m - 12:00 p.m

INSTRUCTIONS

Please answer questions in the exam booklets provided.
This is a closed book exam
No Calculators are permitted.
Use of a regular and/or translation dictionary is permitted.

This exam comprises the cover page, two pages of 8 questions and 1 page of tables .

EXAM IS PRINTED DOUBLE-SIDED.

[Marks]

[10] (1) There is a possibility that a bridge might collapse if an engineer's design is faulty or not. If the design is faulty, the probability that the bridge will collapse is 0.9. If the design is not faulty, the probability of collapse is 0.3. It is also known that the engineer's design is faultless in 80% of cases. If a bridge designed by this engineer does collapse, what is the probability that the design was faulty?

[10] (2) A balanced die is rolled ten times each by five different people. What is the probability that at least two of these people get exactly 4 twos, 3 fours, and 3 sixes?

[10] (3) A certain machine discharges dye into paint cans. The amount in millilitres (ml) of dye discharged is normally distributed with mean μ and variance $\sigma^2 = .16$. If more than 6 ml of dye is discharged when making blue paint, the result is unacceptable. Determine the setting for μ so that only 1% of cans of blue paint will be unacceptable.

[15] (4) The density function of a random variable X is given by

$$f(x) = \begin{cases} kxe^{-4x^2} & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find

i. the value of k,
ii. the distribution function of X,
iii. $P\{2 < X < 4\}$,
iv. $P\{X < 4 | X > 2\}$,
v. $E(X)$.

(b) Suppose that X represents the thickness (in micrometres) of a certain type of microchip produced by Intel Corporation at its plant in Dublin, and that microchips whose thickness is less than 2 or greater than 4 must be rejected. What is the probability that in a batch of 5 of these microchips, at least 2 must be rejected?

[15] (5) Random variables X and Y have joint probability function given in the following table:

		X		
		-1	0	1
	0	.1	.2	.1
Y	1		.2	.1
	2	.2		.1

Find

(a) the marginal probability function of Y,
(b) $E(XY^2)$,
(c) $P\{X + Y < 2\}$,
(d) $E(X|Y = 1)$.

Are X and Y independent? Justify your assertion.

[10] (6) Let X and Y be independent random variables with distributions $N(-1, 9)$ and $N(0, 4)$ respectively. Define

$$Z = X + 3Y$$
$$W = X - 3Y.$$

Find

(a) the distribution of Z,

(b) the correlation coefficient between Z and W.

(c) the correlation coefficient between W and Z.

[20] (7) Suppose that X and Y are random variables with joint density function

$$f(x,y) = \begin{cases} k(3x+y) & \text{if } 0 < x < 2,\ 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find k,

(b) Find the marginal density function of X,

(c) Find $E(Y^2|X=1)$,

(d) Find $P\{X/Y < 2\}$,

(e) Find $\text{Cov}(X,Y)$,

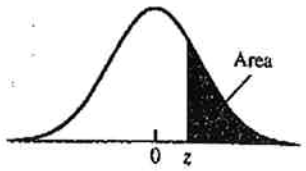
(f) Are X and Y independent? Justify your answer.

[10] (8) The weight in centigrams (1 centigram= .01 gram) of a type of paper clip manufactured by a certain company is a random variable with density function

$$f(x) = \frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^{\alpha}}, \quad x > 0$$

where $\alpha = 25$ and $\beta = 2$. What is the probability that 100 of these paper clips will exceed the capacity of a standard packing carton, which is sixty grams? (Hint: Recall that if $X \sim \Gamma(\alpha,\beta)$, then $E(X) = \alpha\beta$ and $\text{Var}(X) = \alpha\beta^2$.)

Table 4. Normal curve areas
Standard normal probability in right-hand tail
(for negative values of z areas are found by symmetry)



	Second decimal place of z									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000 233									
4.0	.000 031 7									
4.5	.000 003 40									
5.0	.000 000 287									

From R. E. Walpole, *Introduction to Statistics* (New York: Macmillan, 1968).