McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 316

COMPLEX VARIABLES

Examiner: Professor K. GowriSankaran
Associate Examiner: Professor R. Vermes

Date: Wednesday December 9, 2009.
Time: 9:00 a.m. - 12:00 p.m.

INSTRUCTIONS

1. Please answer questions in the exam booklets provided.
2. This is closed book exam.
3. Calculators are not permitted.
4. Dictionaries are not permitted.

This exam comprises the cover page, and 1 pages of 7 questions.

1. Suppose f is an entire function. Prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(|f(z)|^2\right)=4\,|f'(z)|^2.$$

2. Locate all the singular points of

   (a) $\sin(z+\frac{1}{z})$

   and

   (b) $\dfrac{z}{e^z-1}$.

   Determine the nature of the singular points. Find the residue at each of the singularities.

3. (a) State the theorem of Liouville.

   (b) Suppose f is an entire function and $|f(z)|\geq K>0$ for all z in the plane. Prove $f\equiv$ constant.

4. Prove that $z^6+7z+1=0$ has six rootes inside $|z|=2$, five of which lie in $1\leq|z|\leq 2$.

5. Use residue calculus to evaluate $\displaystyle\int_0^\infty \frac{(\cos x)^2}{1+x^2}\,dx$. Justify your steps.

6. Find the bilinear transformation T such that $T(-1)=0$, $T(i)=1$ and $T(1)=\infty$. Find the image of the region given by $|z-1|<2$ and $|z+1|<2$ under T.

7. Use an appropriate Bromwich contour to find the inverse Laplace transformation of $\dfrac{1}{s^2+s+1}$. Justify your steps.