McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 315

ORDINARY DIFFERENTIAL EQUATIONS

Examiner: Professor J. Hurtubise
Associate Examiner: Professor G. Schmidt

Date: Monday December 11, 2006
Time: 2:00PM - 5:00PM

INSTRUCTIONS

1. Please answer all questions in the exam booklets provided.
2. This is a closed book exam.
3. Calculators are not permitted.
4. Use of a regular and/or translation dictionary is not permitted.
5. This exam comprises the cover page, and 1 page of questions and 1 page of the table of Elementary Laplace Transforms.

MATH 315, FINAL EXAMINATION

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1. Solve the following differential equations, with initial conditions, if given:

a. (10 pts) $y' = \frac{x^2+3y^2}{2xy}$ $y(1) = 1$

b. (10 pts) $(x+2)sin(y)dx + x\ cos(y)dy = 0$

c. (10 pts) $y'' + 4y' + 4y = x^{-2}e^{-2x}$

d. (10 pts)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

e. (10 pts) $y''' - y'' - y' + y = 2e^{-x} + 3$, given that one solution of the homogeneous equation is $y = e^x$.

f. (10 pts) $(3x+1)y'' - (9x+6)y' + 9y = 0$, given that $y_1(x) = e^{3x}$ is a solution.

2. Solve in a series centred at $x = 0$. You must first decide whether to use a regular power series or a Frobenius series. Discuss the radius of convergence of the solutions.

a. (10 pts) $y'' - 2xy' + \lambda y = 0$, where λ is a constant.

b. (10 pts) $2x^2y'' + 3xy' + (2x^2 - 1)y = 0$

3. a. (5 pts) Compute the inverse Laplace transform of

$$\frac{2(s-1)e^{-2s}}{s^2 - 2s + 2}$$

b. (5 pts) Compute the Laplace transform of the square wave f

$$f(x) = 1 + \sum_{k=1}^{\infty}(-1)^k u_k(x)$$

c. (10 pts) Solve the initial value problem

$$y'' + 2y' + 2y = cos(x) + \delta(x - \pi/2), \quad y(0) = 0,\ y'(0) = 0$$

Solve the same problem, but with initial conditions $y(0) = 1,\ y'(0) = 0$.

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\frac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\frac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. t^n; $\quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
4. $t^p, \ p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Ex. 6
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a \rvert$	Sec. 6.1; Prob. 8
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a \rvert$	Sec. 6.1; Prob. 7
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}$, $n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t) f(t-c)$	$e^{-cs} F(s)$	Sec. 6.3
14. $e^{ct} f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 19
16. $\int_0^t f(t-\tau) g(\tau)\, d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 28