1. Solve the following equations:

   (a) $\dfrac{dy}{dx} = ay - by^3 \quad (a > 0,\ b > 0)$ Hint: Let $v = y^\alpha$. (Bernoulli equation).

   (b) $\dfrac{dy}{dx} = \dfrac{y^3}{1 - 2xy^2}, \quad y(0) = 1.$

2. (a) Find all values of α for which **all solutions** of

$$x^2y'' + \alpha xy' + \frac{5}{2}y = 0$$

   approach zero, as $x \to \infty$.

   (b) Using the method of reduction of order, show that if $r(r-1) + \alpha r + \beta = 0$ has a double root r_1, then x^{r_1} and $x^{r_1}\ln x$ are the solutions of the Euler equation:

$$x^2y'' + \alpha xy' + \beta y = 0.$$

3. For each of the following equations

$$x(x+3)^2y'' - 2(x+3)y' - xy = 0, \quad xy'' + y' - y = 0.$$

   (a) Find all the regular singular points.

   (b) Determine the indicial equation and the exponents for each regular singular point.

   (c) For the equation $xy'' + y' - y = 0$ only, derive the first three nonzero terms in each of two independent solutions about $x = 0$.

4. Solve the following IVP

$$\begin{cases} y'' - 2y' + 2y = e^{-t} \\ y(0) = 0 \\ y'(0) = 1. \end{cases}$$

   (a) using the method of variation of parameters,

   (b) using the Laplace transform method.

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-315A

ORDINARY DIFFERENTIAL EQUATIONS

Examiner: Professor J.J. Xu
Associate Examiner: Professor

Date: Wednesday, December 15, 1999
Time: 9:00 A.M. - 12:00 Noon.

INSTRUCTIONS

Calculators are neither needed nor permitted.

This exam comprises the cover, one page of questions and one page of Laplace transforms.