

1. Find the solution for the equation

$$\frac{dy}{dx} = \frac{x + 3y - 5}{x - y - 1} .$$

2. Find an integrating factor for the following equation and solve it:

$$ydx + (2xy - e^{-2y})dy = 0 .$$

3. Find the general solution by using the method of variation of parameter for the equation

$$y'' + y' = \tan t \quad \left(-\frac{\pi}{2} < t < \frac{\pi}{2} \right) .$$

4. Find all the regular singular points for following each equation; determine the indicial equation and the exponents at the singularity for each regular singular point.

(a) $x^2y'' + \frac{1}{2}(x + \sin x)y' + y = 0$.

(b) $(x + 1)^2y'' + 3(x^2 - 1)y' + 3y = 0$.

5. Show that the following equation has a regular singular point $x = 0$; find the exponents at this singular point and find the first three non-zero terms of its two independent solutions:

$$2x^2y'' + 3xy' + (2x^2 - 1)y = 0 .$$

6. Solve the following initial value problems by using the Laplace Transformation method

(i)
$$\begin{aligned} y'' + y' + \frac{5}{4}y &= g(t) \\ y(0) = 0, \quad y'(0) &= 1 \end{aligned} \quad g(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0 & t > 0 \end{cases}$$

(ii)
$$\begin{cases} y'' + 2y' + 2y &= \cos t + \delta\left(t - \frac{\pi}{2}\right) \\ y(0) &= 0, \\ y'(0) &= 0. \end{cases}$$

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-315A

ORDINARY DIFFERENTIAL EQUATIONS

Examiner: Professor J.J. Xu
Associate Examiner: Professor J. Loveys

Date: Friday, December 5, 1997
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

NO CALCULATORS PERMITTED

This exam comprises the cover, 1 page of questions and 1 page of Laplace transforms.