1. (10 marks) Determine for what values of α and β (if any) the system

$$\begin{array}{rcrcrl} & & \alpha x_2 & + & x_3 = & \beta \\ \alpha x_1 & & & + & \beta x_3 = & 1 \\ \alpha x_1 & + & \alpha x_2 & + & 2x_3 = & 2 \end{array}$$

possesses the following:

   (a) a unique solution, i.e. a point
   (b) a one-parameter solution, i.e. a line
   (c) a two-parameter solution, i.e. a plane
   (d) no solution.
   (e) Solve the system for (b) and (c) above.

2. (8 marks) Given

$$x(\pi - x) = \frac{8}{\pi}\left[\frac{\sin x}{1^3} + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \cdots\right], \qquad 0 < x < \pi$$

obtain a numerical value for $\sum_{n=1}^{\infty} \frac{1}{n^6}$

3. (14 marks) Find the electric potential distribution inside the quarter circle $0 \le r < a$ if the straight edges are insulated and the potential along the curved edge is $\sin\theta$. Leave your answer in simplest form.

   Hints: (i) Solve $\nabla^2\psi(r,\theta) = 0;\ 0 \le r < a,\ 0 < \theta < \pi/2$
   $\psi_\theta(r,0) = 0,\ \psi_\theta(r,\pi/2) = 0,\ \psi(a,\theta) = \sin\theta.$

   (ii) $$\nabla^2\psi(r,\theta) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}.$$

   (iii) $\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)].$

4. (a) (9 marks) The moment of inertia matrix is given by

$$\begin{bmatrix} 4 & -1 & 1 \\ -1 & 4 & -1 \\ 1 & -1 & 4 \end{bmatrix}$$

   when referred to the xyz coordinate system. Find the principal moments of inertia and unit vectors along the three principal axes.

   (b) (4 marks) Use matrix methods to identify and sketch

$$4x_1^2 + 4x_2^2 + 4x_3^2 - 2x_1x_2 + 2x_1x_3 - 2x_2x_3 = 18.$$

   (c) (5 marks) Obtain the matrix of transformation from the (x_1, x_2, x_3) axes to those axes with respect to which the equation has no cross terms, making certain that this transformation is a rotation. Explain. Indicate how to obtain the angle and axis of rotation.

   (d) (4 marks) Does

$$\int^{\infty}\int^{\infty}\int^{\infty} e^{-[4x^2-2xy+4y^2+2xz+4z^2-2yz]}dxdydz$$

6. (10 marks) Consider the system of vibrating masses below where x_1 and x_2 are measured from their respective equilibrium positions

   (a) Find the normal frequencies of vibration.

   (b) Find the normal modes of vibration.

   (c) If the system has initial displacement $X_0 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ and initial velocity $\dot{X}_0 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ determine the subsequent motion. Assume that there is no friction in the system.

   Hint: The equations of motion are given by:

$$\begin{aligned} 2\ddot{x}_1 &+ 13x_1 - 5x_2 = 0 \\ 2\ddot{x}_2 &- 5x_1 + 13x_2 = 0 \end{aligned}$$

7. (15 marks) The circuit below is governed by the following system of differential equations

$$\frac{dI}{dt} = -\frac{1}{2}I - \frac{1}{8}V + \frac{1}{2}J(t)$$

$$\frac{dV}{dt} = 2I - \frac{1}{2}V,$$

   where I is the current through the inductance, V is the voltage drop across the capacitor, and $J(t)$ is the current supplied by the external source.

   Find $I(t)$ and $V(t)$ if $I(0) = 2$ amperes $V(0) = 3$ volts and $J(t) = e^{-t/2}$.

8. (12 marks) Solve

McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-270B

APPLIED LINEAR ALGEBRA

Examiner: Professor C. Roth
Associate Examiner: Professor D. Sussman

Date: Wednesday, April 28, 1999
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Faculty standard calculators are permitted.

This exam comprises the cover and 2 pages of questions.