MARKS

(8) 1. (a) Given

$$x(\pi - x) = \frac{8}{\pi}\left[\frac{\sin x}{1^3} + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \cdots\right], \quad 0 < x < \pi$$

obtain a numerical value for $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^6}$.

(10) (b) Solve the initial-boundary value problem: $\dfrac{1}{\alpha^2}\dfrac{\partial \psi}{\partial t} = \dfrac{\partial^2 \psi}{\partial x^2}$; $t > 0,\ 0 < x < \pi$

$$\psi(0,t) = 0,\ \psi(\pi,t) = 0,\ \psi(x,0) = x(\pi - x)$$

and interpret physically. It is sufficient to write down explicitly the first three terms of your series solution.

(11) 2. (a) A dielectric material is characterized by the permittivity matrix (tensor) given by

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

when referred to the xyz coordinate axes. Find the principal dielectric constants and unit vectors along the principal axes.

(4) (b) Use matrix methods to identify and sketch

$$2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_1x_3 + 2x_2x_3 = 1 \ .$$

(3) (c) Letting $y_i,\ i = 1,2,3$ denote the axes with respect to which the equation has no cross terms, obtain the matrix of transformation from the x_i to the y_i axes.

(1) (d) Find the cosine of the angle of rotation corresponding to the above transformation.

(5) 3. (a) Does $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp[-(4x^2+2y^2+4z^2+4xy+4xz+4yz)]dxdydz$ converge? If it does, find its value.

(4) (b) The function $f(x,y,z) = 2x^2 + y^2 + 2z^2 + 2xy + 2xz + 2yz + x - 3z - 5$ has an extremum at $\left(-\dfrac{1}{2}, -1, \dfrac{3}{2}\right)$. Find the nature of that extremum. Justify carefully your conclusion.

(9) 4. Determine a unitary transformation that diagonalizes the Hermitian form

$$X^\dagger \begin{bmatrix} 1 & 4+2i \\ 4-2i & 2 \end{bmatrix} X \ .$$

(a) Find the normal frequencies of vibration.
(b) Find the normal modes of vibration.
(c) If the system has initial displacement $X_0 = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ and initial velocity $\dot{X}_0 = \begin{bmatrix} \gamma \\ \delta \end{bmatrix}$ determine the subsequent motion. Assume that there is no friction in the system. Leave your final answer in terms of systems of linear equations.
<u>Hint</u>: The vibrating masses satisfy the following system of differential equations

$$\begin{aligned} M\ddot{x}_1 &= -Kx_1 + K(x_2 - x_1) \\ M\ddot{x}_2 &= -Kx_2 + K(x_1 - x_2) \ . \end{aligned}$$

(12) 6. The circuit below is governed by the following system of differential equations:

$$\begin{aligned} \frac{dI}{dt} &= -\frac{1}{2}I - \frac{1}{8}V + \frac{1}{2}J(t) \\ \frac{dV}{dt} &= 2I - \frac{1}{2}V, \end{aligned}$$

where I is the current through the inductance, V is the voltage drop across the capacitor, and $J(t)$ is the current supplied by the external source.

(a) Find $I(t)$ and $V(t)$ if $I(0) = 2$ amperes $V(0) = 3$ volts and $J(t) = 0$ using the exponential matrix method.
(b) Write down the Green's matrix for the system and discuss its significance.

(12) 7. Solve

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{2}{t^2} & \dfrac{2}{t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} t^2 \\ 2t \end{bmatrix}; \ t \geq 1 \text{ with } X(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} .$$

(5) 8. (a) Obtain the critical points of the system and discuss their nature and stability

$$\begin{aligned} \dot{x} &= 2y + x^2 \\ \dot{y} &= -2x - 4y \ . \end{aligned}$$

(5) (b) By using a Liapounov function of the form $Ax^{2n} + Cy^{2m}$ discuss the stability of

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-270B

APPLIED LINEAR ALGEBRA

Examiner: Professor C. Roth
Associate Examiner: Professor D. Sussman

Date: Friday, April 18, 1997
Time: 2:00 P.M. - 5:00 P.M.

This exam comprises the cover and 2 pages of questions.