1. Evaluate $\iint\limits_{D} (x^4 - y^4)dA$, where D is the part of the first quadrant with

$$1 \le x^2 - y^2 \le 3, \qquad 2 \le xy \le 3.$$

2. Find the coordinates of the centroid of the region inside the sphere $x^2 + y^2 + z^2 = a^2$, outside the cone $x^2 + y^2 = b^2 z^2$, and above the xy-plane.

3. Let C be the curve that goes from $(-1,0)$ to $(0,1)$ along the graph of $y = 1 - x^2$, and then from $(0,1)$ to $(1,0)$ along the graph of $y = 1 - x$. Compute $\int_C ydx - xdy$.

4. Find the area of that part of the surface $z = 2x^2 + 3xy - 2y^2$ that is inside the cylinder $25x^2 + 25y^2 = 3$.

5. (a) If $\vec{F} = \dfrac{2x\vec{i} + 2y\vec{j} + 2z\vec{k}}{(x^2 + y^2 + z^2)^{3/2}}$, compute div $\vec{F}((x,y,z) \ne (0,0,0))$.

   (b) If $\vec{F}$ is as in part (a), and S is the spherical surface $x^2 + y^2 + z^2 = a^2$ with normal directed outward, compute $\iint\limits_{S} \vec{F} \cdot \hat{N}dS$.

   (c) If $\vec{F}$ is as in part (a), and S is the surface of the cube $-1 \le x \le 1$, $-1 \le y \le 1$, $1 \le z \le 3$ with normal directed outward, compute $\iint\limits_{S} \vec{F} \cdot \hat{N}dS$.

6. Let $\vec{F} = y^3\vec{i} - 2z^3\vec{j} + x^3\vec{k}$.
   If C is the curve of intersection of $x^2 + y^2 + z^2 = 1$ and $z = x$, directed counterclockwise when viewed from above, compute $\int_C \vec{F} \cdot d\vec{r}$ using Stokes's theorem.

7. The equations

$$xyuv = -2$$
$$x^2 + 2y^2 + 3u^2 + 4v^2 - uv = 17$$

   define u, v as functions of x, y in the neighbourhood of the point $x = 1$, $y = -1$, $u = 2$, $v = 1$. Find $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial v}{\partial x}, \dfrac{\partial v}{\partial y}$ in the neighbourhood of that point.

8. Use the method of Lagrange multipliers to find the maximum value of z on the intersection of $x^2 + 2y^2 + 3z^2 = 1$ and $x + y + z = 0$.

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-265B

ADVANCED CALCULUS

Examiner: Professor D. Sussman   Date: Friday, April 18, 1997
Associate Examiner: Professor J.J. Xu   Time: 9:00 A.M. - 12:00 Noon

INSTRUCTIONS

Calculators Not Permitted

This exam comprises the cover and 1 page of questions.