McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATH 264

ADVANCED CALCULUS

Examiner: Professor W. Jonnson
Associate Examiner: Dr. Francis Valiquette
Francis Valiquette

Date: Thursday December 17, 2009
Time: 9:00 AM- 12:00 PM

INSTRUCTIONS

1. Please answer questions in the exam booklets provided.
2. This is a closed book examination. No notes, books or any other aids are allowed.
3. Calculators are not permitted.
4. Translation dictionaries are permitted.

This exam is comprised of the cover page, and 1 page of 7 questions.

Math 264 – Advanced Calculus – Final Exam
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1. Evaluate the double integral

$$\iint_R x^2y^3\,dxdy$$

where R is the region bounded by the curves

$$y = 1/x, \quad y = 4/x, \quad y^2 = 1/x, \quad \text{and} \quad y^2 = 4/x.$$

2. Evaluate the line integral of

$$\mathbf{F} = (ye^{xy} + \cos(x+y) + yz)\,\mathbf{i} + (xe^{xy} + \cos(x+y) + xz)\,\mathbf{j} + xy\,\mathbf{k}$$

along the spiral

$$\mathbf{r}(t) = t\cos(t)\,\mathbf{i} + t\sin(t)\,\mathbf{j} + t\,\mathbf{k}, \qquad 0 \le t \le 2\pi.$$

3. Evaluate the line integral

$$\oint_{C_1 \cup C_2} -x^3dy + y^3dx$$

where C_1 is the circle of radius 1 centered at the origin travelled counterclockwise and C_2 is the circle of radius 2 centered at the origin travelled clockwise.

4. **a)** State Gauss' Theorem (also known as the Divergence Theorem).

   b) Let $\mathbf{F} = xy^2\,\mathbf{i} + yz^2\,\mathbf{j} + x^2z\,\mathbf{k}$ and $\mathcal{S}$ be the sphere of radius 1 centered at the origin. Orient $\mathcal{S}$ with the outward pointing normal vector. Evaluate

$$\oiint_{\mathcal{S}} \mathbf{F} \bullet \hat{\mathbf{n}}dS.$$

5. Let $\mathcal{S}$ be the surface given by $z = 1 - x^2$ with $0 \le x \le 1$ and $-2 \le y \le 2$ oriented upward. Verify Stokes' Theorem for $\mathbf{F} = xy\,\mathbf{i} + yz\,\mathbf{j} + xz\,\mathbf{k}$ and $\mathcal{S}$ by computing

   a) $\iint_{\mathcal{S}} \nabla \times \mathbf{F} \bullet \hat{\mathbf{n}}dS$.

   b) $\oint_C \mathbf{F} \bullet d\mathbf{r}$, where C is the corresponding oriented boundary of $\mathcal{S}$.

6. Solve the wave equation

$$u_{tt} = u_{xx}$$

with boundary conditions

$$u(0,t) = u(5,t) = 0, \quad u(x,0) = 0, \quad u_t(x,0) = 3\sin(2\pi x) - 2\sin(5\pi x).$$

7. **a)** Find the half-range Fourier sine series of

$$f(x) = \begin{cases} x & 0 \le x \le \frac{1}{2}, \\ 1 - x & \frac{1}{2} < x \le 1. \end{cases}$$

   b) Use the method of separation of variables to solve the heat equation

$$u_t = u_{xx}, \qquad 0 \le x \le 1, \qquad t \ge 0,$$

   with boundary conditions

$$u(0,t) = u(1,t) = 0, \qquad u(x,0) = f(x) = \begin{cases} x & 0 \le x \le \frac{1}{2}, \\ 1 - x & \frac{1}{2} < x \le 1. \end{cases}$$