Name:
Student ID Number

McGill University
Faculty of Engineering

Final Examination: Math 262
Intermediate Calculus

Examiner: Prof. N.Sancho   Date: Thursday, Dec.13, 2007
Assoc. Examiner: Dr. D. Serbin   Time: 9:00 – 12:00

Instructions

Attempt all questions. No calculators allowed. No dictionaries are permitted. This is a closed book exam. Answer all questions on the pages provided. If needed continue questions on PRECEDING page or in the blank pages at the end of the booklet.

This examination comprises the cover and 12 pages with 9 questions and 3 blank pages

1	2	3	4	5	6	7	8	9

Final Math 262 Fall 2007 – Dec. 13th – 9:00 to 12:00

1. (10 marks) Find the interval of convergence of the power series including the end-points:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^n (x-3)^n}{n}.$$

2(a) (6 marks) Find the Maclaurin series for the function

$$E(x) = \int_0^x \frac{e^t - 1}{t}\,dt.$$

How many terms of this series are needed to compute E(1) with error less than 0.001?

(b) (6 marks) Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{1}{ne^n}.$$

3. (12 marks) Find the general solution of $\frac{d^2 y}{dx^2} = xy$, in the form of a power series $y = \sum_{n=0}^{\infty} a_n (x-1)^n$ with a_0 and a_1 arbitrary.

4. A curve is given by $\mathbf{r}(t) = (\cos t + t\sin t)\,\mathbf{i} + (\sin t - t\cos t)\,\mathbf{j} + t^2\,\mathbf{k}$.
   (a) (4 marks) Find the arc length between the points where $t = 0$ and $t = \pi/2$.
   (b) (4 marks) Find the unit tangent vector $\widehat{\mathbf{T}}$, unit normal $\widehat{\mathbf{N}}$, and unit binormal $\widehat{\mathbf{B}}$ at the point $t = \pi/2$.
   (c) (4 marks) Find the curvature $\mathbf{K}$ at the point $t = \pi/2$.

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5 (a) (5 marks) Find the parametric equation of the tangent line to the curve of intersection of the surfaces $x^2 + y^2 = 1$ and $x + y + z - 1 = 0$ at the point (1, 0, 0).

(b) (5 marks) Let f(x, y) = ln $|\mathbf{r}|$ where $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$. Show that $\nabla f = \dfrac{\mathbf{r}}{|\mathbf{r}|^2}$.

6 (a) (6 marks) Find the direction in which the function $f(x, y, z) = x^3 z + y^3 z^2 - xyz$ has its maximum rate of increase at the point (1, 1, 1). What is the rate of increase at (1, 1, 1) in the direction of the point (1, 2, 2)?

(b) (4 marks) Find the tangent plane to the surface: $x^3 z + y^3 z^2 - xyz = 1$ at the point (1, 1, 1).

7. (12 marks) Let x, y, z, u, v be related by the equations

$$x = u + \ln v$$

$$y = v - \ln u$$

$$z = 2u + v$$

Find $\left(\frac{\partial z}{\partial x}\right)_y$ and $\left(\frac{\partial z}{\partial y}\right)_x$ at u = 1, v = 1.

8. (12 marks) Find and classify the critical points of the function

$$f(x,y) = x^4 + y^4 - (x+y)^2.$$

9. (10 marks) Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = xyz$ on the sphere $x^2 + y^2 + z^2 = 12$.

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