NOTE TO PRINTER

McGILL UNIVERSITY
FACULTY OF SCIENCE
FINAL EXAMINATION

MATHEMATICS 189–240A

DISCRETE STRUCTURES AND COMPUTING

EXAMINER: Professor W. G. Brown   DATE: December 12th, 1996
ASSOCIATE EXAMINER: Professor W. O. J. Moser   TIME: 14:00 – 17:00 hours

SURNAME: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

MR, MISS, MS, MRS, &c.: ☐☐☐

GIVEN NAMES: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

STUDENT NUMBER: ☐☐☐☐☐☐☐☐ COURSE AND YEAR: ☐☐☐☐☐

Instructions

1. Fill in the above clearly.

2. Do not tear pages from this book; all your writing — even rough work — must be handed in.

3. Calculators are not permitted.

4. This examination booklet consists of this cover, Pages 1 through 9 containing questions; and Pages 10 and 11, which are blank.

5. Show all your work. All solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you <u>must</u> indicate any continuation clearly on the page where the question is printed!

6. You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a)	1(b)	2	3	4	5(a)	5(b)	5(c)
/7	/3	/10	/10	/10	/5	/5	/5
6(a)	6(b)	7(a)	7(b)	8(a)	8(b)	9(a)	9(b)
/5	/5	/5	/5	/7	/3	/5	/5

1. (a) [7 MARKS] Let $f : A \to B$ be a function. Show carefully that, if f is an injection, and S and T are subsets of A,

$$f(S \cap T) = f(S) \cap f(T) \,. \tag{1}$$

(b) [3 MARKS] Show that (**??**) need not hold if f is not an injection.

2. [10 MARKS] A simple undirected graph $G = (V, E)$ (i.e., an undirected graph $G = (V, E)$ without loops or multiple edges) has the property that its chromatic number is 3; but that, after any edge is removed, the resulting graph has chromatic number 2. Showing all your work, determine all graphs G with this property.

3. [10 MARKS] An examination has 5 problems, on each of which a student can obtain a grade between 0 and 3 inclusive. Using generating functions — no other method will be accepted here — determine the number of different ways in which a student can obtain a grade of 9.

4. [10 MARKS] Using any method studied in this course, solve the recurrence $a_{n+1} = 2a_n + 3a_{n-1}$, $(n \geq 1)$, subject to initial conditions $a_0 = 1$, $a_1 = 7$.

5. (a) [5 MARKS] Determine the number of different strings that can be formed from all the letters of the word PEPPERCORN.

   (b) [5 MARKS] Determine the number of different strings that can be formed from all the letters of the word PEPPERCORN where the letters C and N cannot be side by side (in either order), and where O cannot appear immediately to the left of N.

   (c) [5 MARKS] Determine the number of different strings that can be formed from all the letters of the word PEPPERCORN where no two P's can appear side by side.

6. (a) [5 MARKS] Prove or disprove: If (P, R) and (Q, S) are posets with $|P| = |Q| = 4$, and if $|R| = |S|$, then there exists a bijection $f : P \rightarrow Q$ such that

$$\forall p_1 \in P \ \left[\forall p_2 \in P \ [((p_1, p_2) \in R) \Leftrightarrow ((f(p_1), f(p_2)) \in S)] \right]$$

(b) [5 MARKS] Prove or disprove: On the set $\{1, 2, 3\}$ there is no equivalence relation R for which $|R| = 6$.

7. (a) [5 MARKS] Prove or disprove: There exist at least 2 non-isomorphic graphs on 8 vertices whose degrees are 2, 2, 2, 2, 3, 3, 3, 3.

   (b) [5 MARKS] Prove or disprove: There exist at least 2 non-isomorphic trees on 10 vertices whose degrees are 1, 1, 1, 1, 1, 2, 3, 3, 3, 4.

8. (a) [7 MARKS] Show that, if 5 distinct integers are selected from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8\},$$

   then there must be a pair of these integers whose sum is equal to 9.

   (b) [3 MARKS] Show that the preceding statement fails if only 4 distinct integers are selected from the set.

9. (a) [5 MARKS] Let p be a positive prime integer. Describe in detail an algorithm by which one can find, for each integer a which is not divisible by p, integers b and c such that
$$ab + pc = 1\,.$$

(b) [5 MARKS] Show that 127 is prime by dividing it by certain positive integers less than 12. Explain why your method works.

(c) [5 MARKS] Use the method you have described in (a) to determine, for the integer 15, an integer ℓ such that

$$15\,\ell \equiv 0 \pmod{127}\,.$$

CONTINUATION PAGE FOR PROBLEM NUMBER

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