STUDENT NAME: ____________________________

STUDENT NUMBER: _______________

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 222

CALCULUS III

Examiner: W. Jonsson
Associate Examiner: N. Sancho

Date: Thursday, April 18, 2011
Time: 9:00 AM - 12:00 PM

Instructions

1. Total number of points: 92. Your total will be pro-rated to be out of 85 for the final grade calculation.
2. No books, calculators or notes are allowed for the exam. Do not rip pages from the examination book.
3. There are 4 versions of this examination. **This version belongs to Group 1**.
4. Answers to the questions in Part I, each of which is worth 3 points, are to be entered on the machine readable sheet with a soft lead pencil. There are fourteen such questions for a total of 42 points
5. Answers to the questions in part II, each of which is worth 10 points, are to be written in the space provided on the examination paper. There are five such questions for a total of fifty points.
6. Your answers may contain π or other expressions that cannot be computed without a calculator, e.g. $\ln 2$, $300^{1/2} + 13 \cdot 150^{-3/2}$.
7. All material (question papers, machine readable sheets) must be turned in.
8. Name, Student number and group number of your examination **MUST** be entered on the question paper and on the machine readable sheet.

Score Table

Part I Multiple Choice	
Part II Problems	Points
1.	
2.	
3.	
4.	
5.	
Total:	

This exam comprises eight pages, including the cover and four blank pages for further work.
The Examination Security Monitor Program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, wither to initiate or corroborate and investigation or a charge of cheating under section 16 of the Code of Student Conduct and Disciplinary procedures.

PART I. **Multiple choice questions**

1. Of the two infinite series

$$A : \sum_{n=1}^{\infty} e^{-n}\sin^2 n \qquad B : \sum_{n=1}^{\infty} \frac{1}{n}\sqrt{1-\frac{1}{n}}$$

   (a) both are divergent
   (b) both are absolutely convergent
   (c) A is divergent and B is convergent
   (d) A is convergent and B is divergent
   (e) A is absolutely convergent and B is conditionally convergent

2. The coefficient of x^4 in the Maclaurin series for $\dfrac{1}{1-\sin(x^2)}$ is
   (a) $-1/2$, (b) 0, (c) $1/2$, (d) 1, (e) -1

3. Let s denote the sum of the alternating series $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \cdots + \dfrac{(-1)^n}{2n+1} + \cdots$ then
   (a) $\frac{2}{3} < s < 7/8$, (b) $s < 0$, (c) $0 < s < 2/3$, (d) $s > 1$, (e) $7/8 < s < 1$.

4. The power series $\sum_{n=1}^{\infty} \dfrac{(x+1)^{6n}}{n^2 8^n}$ has interval of convergence

   (a) $1-\sqrt{2} \le x \le 1+\sqrt{2}$
   (b) $-1-\sqrt{2} < x < -1+\sqrt{2}$
   (c) $-1-\sqrt{2} \le x \le 1-\sqrt{2}$
   (d) $-1-\sqrt{2} \le x \le -1+\sqrt{2}$
   (e) $-1-\sqrt{2} \le x \le 1+\sqrt{2}$

5. The arc length L of the curve $\vec{R} = \left(\dfrac{2}{3}t^3, t, t^2\right)$ from the origin to its intersection with the plane $x = 18$ satisfies

   (a) $23.5 \le L \le 24.5$
   (b) $22.5 \le L < 23.5$
   (c) $21.5 \le L < 22.5$
   (d) $20.5 \le L < 21.5$
   (e) $19.5 \le L < 20.5$

6. Let $z(x, y)$ be defined implicitly by $z^3 + z^2x + zy = 5$. Then $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y}$ at the point $(3, 1, 1)$ has the value
   (a) -2, (b) -0.2, (c) 0, (d) 0.2, (e) -20.

7. The unit tangent $\vec{T}$ to the curve $\vec{R} = \left(\dfrac{2}{3}t^3, t, t^2\right)$ when $t = 3$ is parallel to

   (a) $12\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}k$
   (b) $18\mathbf{i} + \mathbf{j} + 6\mathbf{k}k$
   (c) $18\mathbf{i} + 6\mathbf{k}$
   (d) $24\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$
   (e) $18\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

8. The curvature at $(0, 0, 0)$ on the curve $\vec{R} = \left(\dfrac{2}{3}t^3, t, t^2\right)$ has the value
   (a) 2, (b) 1, (c) 0, (d) 3, (e) 6.

9. The critical point(s) of $F(x,y) = xye^{\left(\frac{-y}{2}-x\right)}$ consist of

(a) one relative minimum and one saddle point
(b) one relative maximum and one saddle point
(c) more than one saddle point
(d) one relative maximum and two saddle points
(e) one saddle point

10. The tangent plane to the surface $z = x\ln(x+2y-3)$ at $(2,1,0)$ is

(a) $z = -2x - 4y + 8,$
(b) $z = 2x + 4y,$
(c) $z = 3x + 4y - 10,$
(d) $z = 2x + 4y - 8,$
(e) $z = 2x + 2y - 6.$

11. The fourth degree Taylor polynomial of $f(x) = x^2e^x$ centered at $a = 0$ is

(a) $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24},$
(b) $1 + x + \frac{x^2}{2} + \frac{x^3}{6},$
(c) $x + x^2 + \frac{x^3}{2} + \frac{x^4}{6},$
(d) $x^2 + x^3 + \frac{x^4}{2},$
(e) $\frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}.$

12. A particle is moving along the trajectory $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + t\,\mathbf{k}$. At time $t = \pi/2$ the velocity vector $\mathbf{v}(\pi/2)$ and the acceleration vector $\mathbf{a}(\pi/2)$ are

(a) $\mathbf{v}(\pi/2) = -2\mathbf{i} + \mathbf{k},$ and $\mathbf{a}(\pi/2) = -3\mathbf{j},$
(b) $\mathbf{v}(\pi/2) = -2\mathbf{j} + \mathbf{k},$ and $\mathbf{a}(\pi/2) = -3\mathbf{i},$
(c) $\mathbf{v}(\pi/2) = 2\mathbf{i} + \mathbf{k},$ and $\mathbf{a}(\pi/2) = 3\mathbf{j},$
(d) $\mathbf{v}(\pi/2) = 2\mathbf{j} + \mathbf{k},$ and $\mathbf{a}(\pi/2) = 3\mathbf{i},$
(e) $\mathbf{v}(\pi/2) = 3\mathbf{i} + \mathbf{k},$ and $\mathbf{a}(\pi/2) = 2\mathbf{j}.$

13. The directional derivative of $z = x\ln(x+2y-3)$ at the point $(2,1)$ in the direction parallel to the vector $(3,4)$ is (a) -4.4, (b) 4.4, (c) -2.8, (d) 2.8, (e) 22.

14. At the point $(2,1)$ the direction in which the function $f(x,y) = \frac{x^2}{4} + y^2$ has the maximum rate of change is given by the vector

(a) $-\mathbf{i} - 2\mathbf{j},$
(b) $-\mathbf{i} + 2\mathbf{j},$
(c) $2\mathbf{i} + \mathbf{j},$
(d) $\mathbf{i} + 2\mathbf{j},$
(e) $\mathbf{i} - 2\mathbf{j}.$

END OF MULTIPLE CHOICE SECTION

Part II: QUESTIONS REQUIRING WRITTEN ANSWERS

1. Find the volume above the XY-plane cut off between the parabolic cylinders

$$y = 1 - x^2,\ y = -1 + x^2,\ \text{and below the surface } z = 4 - x^2 - y^2.$$

2. (a) Estimate the value of $\int_0^{0.1} \frac{1-\cos x}{x^2}\,dx$ correct to 5 decimal places by finding the power series for $(1-\cos x)/x^2$ and integrating.

(b) Using Newton's method, approximate a root of $y = f(x) = x^2 - 5$ using the initial estimate $x_0 = 2$, then estimate the error in this approximation using the remainder formula given in class.

3. If z and w are defined by the equations

$$\begin{aligned} z + w^3 &= x^2 + y^2 \\ 2z^3 + w^2 &= 2x + y, \end{aligned}$$

find $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}, \dfrac{\partial w}{\partial x}, \dfrac{\partial w}{\partial y}$ at the point where $(x, y) = (1, 1)$ and $(z, w) = (1, 1)$.

4. Evaluate

$$\int_{y=0}^{\sqrt{\pi}} \int_{x=y^2}^{\pi} y \sin(x^2) dx\ dy\ .$$

5. Using the method of Lagrange Multipliers (or otherwise) find the maxima and minima of

$$f(x, y, z) = x^2 + y^2 + z^2$$

subject to the two constraint

$$\begin{aligned} x^2 + y^2 + z^2 &= 2z \\ x + y + z &= 1 \; . \end{aligned}$$

Continuation of solution for problem: ______.
You must refer to this page on the page where the problem is printed.

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