1. (a) Find the interval of convergence of the series:

$$\sum_{n=1}^{\infty}(-1)^n \frac{x^{2n}}{n^2 2^n} .$$

(b) If $F(x)$ denotes the sum of the series in (a) for x in its interval of convergence, compute $F(1/2)$ and $\int_0^1 f(x)dx$ to 2 decimal places (i.e. with error < 0.005).

2. (a) Find the length of the part of the curve

$$\vec{r}(t) = (2\sin t,\ 2\cos t, 3t)$$

cut off by the planes $z = 1,\ z = 4$.

(b) Find the equation of the tangent plane to the surface:

$$x^2y + y^2z + z^2x = 1$$

at the point $(1, 1, -1)$.

(c) Find the directional derivative of the function

$$f(x, y, z) = x^3y + y^3z + z^3x$$

at the point $(1, -1, 1)$ in the direction of the maximum rate of increase of the function $2x^3 + 3xy + 4y^2$ at this point.

3. Find the dimensions of an open-topped rectangular box of volume 1m^3 whose surface area is smallest possible.

4. (a) If $f(x, y)$ is differentiable and $g(r, \theta) = f(r\cos\theta, r\sin\theta)$ compute

$$\frac{\partial g}{\partial r}, \frac{\partial g}{\partial \theta}, \frac{\partial^2 g}{\partial r \partial \theta} .$$

(b) Suppose $g(r, \theta)$ is defined as in part (a). Show that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial g}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial g}{\partial \theta}\right)^2 .$$

5. Find and classify the critical points of $f(x, y) = 6xy - x^3 - y^3$.

6. Sketch the domain of integration and then compute

$$\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\, dy\, dx.$$

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-222A

CALCULUS III

Examiner: Professor J. Toth
Associate Examiner: Professor J. Labute

Date: Monday, December 21, 1998
Time: 9:00 A.M. - 12:00 Noon.

INSTRUCTIONS

Each question is worth 10 marks.
No Calculators are permitted.

This exam comprises the cover and 1 page of questions.