FACULTY OF SCIENCE — FINAL EXAMINATION

MATHEMATICS 222 — CALCULUS 3

Examiner: Professor S. W. Drury

Date: Friday, December 17, 2004

Associate examiner: Professor W. Jonsson

Time: 9:00 am. – 12:00 noon

STUDENT NAME: ______________________

STUDENT NUMBER: ____________

VERSION 1

Instructions

1. Identify yourself by writing your name and student number on both this paper and the computer readable sheet. The consequences of writing illegibly should be obvious.
2. This examination paper may not be removed from the examination room by the candidate. All work, including this paper, the computer readable sheet and any answers or rough work in examination booklets must be handed in.
3. Answers to part I (questions 1 thru 10) are to be entered onto the computer readable sheet provided using a soft lead pencil.
4. **This exam is Version 1.** It is important that the version number for the question paper be entered into the column next to the student number on the computer readable sheet. Enter your student number and fill in the appropriate disk below each digit. Fill in the check bits as indicated. Enter your name, the course number (MATH 222) and sign the card.
5. The answers to part II, (questions 11 thru 16) are to be written on the blank part of the question paper below the question.
6. Questions 1 thru 10 are worth 3 points each, questions 11 thru 16 are worth 12 points each. The total number of points available is 102. As a rough guide, at least initially, you should not spend more than 5 minutes on a question in part I nor more than 20 minutes on a question in part II.
7. This examination booklet consists of this cover, pages 2 through 9 containing questions and two further continuation pages.
8. You are expected to show all your work. Write your solution in the space provided below the question. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
9. You are advised to spend the first minute or two scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)
10. The examination Security Monitor program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, either to initiate or corroborate an investigation or a charge of cheating under Section 16 of the Code of Student Conduct and Disciplinary Procedures.
11. This is a closed book exam. No notes, calculators, mobile phones, PDA's etc. are allowed.

PLEASE DO NOT WRITE INSIDE THIS BOX

multiple choice	/30
question 11	/12
question 12	/12
question 13	/12
question 14	/12
question 15	/12
question 16	/12
grand total	/102

Part I — Answer on the computer readable sheet

1. (3 points) When Taylor's Theorem is applied to $(1+4x)^{\frac{1}{2}}$ at $x=0$ one obtains

$$(1+4x)^{\frac{1}{2}} = 1 + 2x + R(x)$$

where the precise form of the Lagrange Remainder $R(x)$ is given by

(A) $-4(1+4x)^{-\frac{3}{2}}u^2$, (B) $-2(1+4u)^{-\frac{3}{2}}x^2$, (C) $-(1+4u)^{-\frac{3}{2}}x^2$,
(D) $-(1+4x)^{-\frac{3}{2}}x^2$, (E) $-4(1+4u)^{-\frac{3}{2}}u^2$.

where u is an unknown number between 0 and x.

2. (3 points) The Maclaurin series of $\ln(1+2x+x^2)$ up to the term in x^3 is

(A) $2x+x^2+\frac{1}{3}x^3$ (B) $2x-x^2+\frac{1}{3}x^3$ (C) $2x+x^2+\frac{2}{3}x^3$
(D) $2x-x^2+\frac{2}{3}x^3$ (E) $2x+x^2-\frac{1}{3}x^3$

3. (3 points) The power series

$$\sum_{n=0}^{\infty} \frac{2^n}{(n+3)} x^{2n}$$

has radius of convergence

(A) 1, (B) $\sqrt{2}$, (C) $\frac{1}{\sqrt{2}}$, (D) $\frac{1}{2}$, (E) ∞.

4. (3 points) The surface $z = x^2 + 2xy + 4y^2$ has a horizontal tangent plane at

(A) $(x,y)=(0,0)$, (B) $(x,y)=(2,-1)$, (C) $(x,y)=(\frac{1}{2},-1)$,
(D) $(x,y)=(-\frac{1}{2},0)$, (E) $(x,y)=(-\frac{1}{2},1)$.

5. (3 points) Let $g(x,y) = f(x^2-2xy)$ with f a differentiable function of one variable, then necessarily

(A) $(x-y)\frac{\partial g}{\partial y} + x\frac{\partial g}{\partial x} = 0$, (B) $(x-y)\frac{\partial g}{\partial y} - x\frac{\partial g}{\partial x} = 0$, (C) $(x-y)\frac{\partial g}{\partial x} + x\frac{\partial g}{\partial y} = 0$,
(D) $(x-y)\frac{\partial g}{\partial x} - x\frac{\partial g}{\partial y} = 0$, (E) $(x+y)\frac{\partial g}{\partial x} - x\frac{\partial g}{\partial y} = 0$.

6. (3 points) The rate of increase of the function $x^2 + yz$ per unit length at the point $(2,2,2)$ in the direction of the point $(1,4,4)$ is

(A) 0, (B) $\sqrt{2}$, (C) $\frac{4}{3}$, (D) $\frac{1}{3}$, (E) 2.

7. (3 points) Consider the Taylor expansion of $g(t) = \int_0^t 3x\sin(x)dx$ about $t = 0$. Rounded to six decimal places, the value of $g(0.1)$ is

(A) 0.000999, (B) 0.000998, (C) 0.001001, (D) 0.001002, (E) 0.000995.

8. (3 points) The volume of the region of three dimensional space given by $0 \leq z \leq 4 - x^2 - y^2$ is

(A) $\frac{16\pi}{3}$, (B) 4π, (C) 8π, (D) $\frac{8\pi}{3}$, (E) 12π.

9. (3 points) The volume of a circular cone of radius r and height h is $\frac{1}{3}\pi r^2 h$. Suppose that the height of the cone is increasing at a rate of 3 cm/sec and that the radius is decreasing at a rate of 2 cm/sec. Then at an instant when the height is 30 cm and the volume is 1000π cubic cm, the volume is

(A) increasing at a rate of 100π cubic cm/sec. (B) increasing at a rate of 300π cubic cm/sec.
(C) decreasing at a rate of 100π cubic cm/sec. (D) decreasing at a rate of 300π cubic cm/sec.
(E) increasing at a rate of 500π cubic cm/sec.

10. (3 points) A region R of three dimensional space is given by the constraints $z^2 \geq x^2+y^2$ and $x^2+y^2+z^2 \leq 4$. The integral

$$\iiint_R f(x, y, z)dxdydz$$

transforms into spherical coordinates as

(A) $\int_{\phi=0}^{\pi/2}\int_{\rho=0}^{2}\int_{\theta=0}^{2\pi} f(x,y,z)\rho^2\sin(\phi)\,d\theta\,d\rho\,d\phi$

(B) $\int_{\phi=0}^{\pi/2}\int_{\rho=0}^{4}\int_{\theta=0}^{2\pi} f(x,y,z)\rho^2(\sin(\phi))^2\,d\theta\,d\rho\,d\phi$

(C) $\int_{\phi=0}^{\pi/2}\int_{\rho=0}^{2}\int_{\theta=0}^{2\pi} f(x,y,z)\rho^2(\sin(\phi))^2\,d\theta\,d\rho\,d\phi$

(D) $\int_{\phi=0}^{\pi/4}\int_{\rho=0}^{2}\int_{\theta=0}^{2\pi} f(x,y,z)\rho^2\sin(\phi)\,d\theta\,d\rho\,d\phi$

(E) $\int_{\phi=0}^{\pi/4}\int_{\rho=0}^{4}\int_{\theta=0}^{2\pi} f(x,y,z)\rho^2(\sin(\phi))^2\,d\theta\,d\rho\,d\phi$

where in the answers, it is to be assumed that $f(x, y, z)$ has been correctly rewritten in terms of ρ, ϕ and θ.

Part II — Answer in the space provided below the question

11. (12 points) Find and classify the critical points of the function $f(x, y) = 2x^3 + 7x^2 - 2xy + y^2$ in the whole (x, y)-plane.

Continue solution opposite

then on page

Exam continued overleaf...

12. (12 points) Suppose that the integral $\int_{y=0}^{3} \int_{x=y^2-2y}^{y} f(x,y)\, dx\, dy$ is expressed as an area integral $\iint_R f(x,y)\, dA$ over a region of R of the plane.

(i) Describe the region R and draw a diagram showing R.

(ii) Express this same integral in terms of one or more iterated integrals in which the order of integration has been reversed.

Continue solution opposite

then on page

Exam continued overleaf...

13. (12 points) Suppose that $z = z(x, y)$ is determined implicitly in terms of x and y by the equation

$$z^3x + zy^2 + 3xy = 5$$

in such a way that $z(1, 1) = 1$. Find $\frac{\partial z}{\partial x}(1, 1)$, $\frac{\partial z}{\partial y}(1, 1)$ and $\frac{\partial^2 z}{\partial x \partial y}(1, 1)$.

Continue solution opposite
then on page

Exam continued overleaf...

14. (12 points) Use cylindrical coordinates to evaluate

$$\iiint_R 2yz\,dx\,dy\,dz$$

where R is the region of 3-space given by the two inequalities $y \geq 0$ and $0 \leq z \leq 1 - x^2 - y^2$.

Continue solution opposite
then on page ☐

Exam continued overleaf...

15. (12 points) Use the method of Lagrange multipliers to find the maximum value of the function $f(x, y, z) = x^4y^3z^2$ on the portion of the plane $8x + 3y + 4z = 18$ in the quadrant $x \geq 0$, $y \geq 0$, $z \geq 0$.

Continue solution opposite
then on page

Exam continued overleaf...

16. (i) (6 points) Find the arclength of the helix $t \mapsto (3t, 4\cos(t), -4\sin(t))$ between the points $(0, 4, 0)$ and $(3\pi, -4, 0)$.

(ii) (6 points) Find the surface area of the portion of the surface $z = xy$ lying above the disk $x^2 + y^2 \leq 1$ in the (x, y)-plane.

Continue solution opposite
then on page

Exam continued overleaf...

CONTINUATION PAGE FOR PROBLEM NUMBER

You *must* refer to this continuation page on the page where the problem is printed!

Continue solution opposite
then on page

Exam continued overleaf…

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Continue solution opposite
then on the back