McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 222

CALCULUS III

Examiner: Professor J. Labute
Associate Examiner: Professor W. Jonsson

Date: Tuesday, December 10, 2002
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Attempt all questions.
Questions are not necessarily of equal value.
Justify all your statements.
Please write answers in exam booklets.
Dictionaries are not permitted.

This exam comprises the cover and 2 pages with 8 questions.

1. (a) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n \log(n+1)}.$$

(b) Find a power series representation about the point $x = 0$ for

$$g(x) = \frac{4}{(1-x)^2}.$$

2. (a) Using a power series expansion for the sine function, compute

$$\int_0^1 \sin(x^2)\, dx$$

to 3 decimal places.

(b) Compute

$$\lim_{x \to 0} \frac{(e^{2x}-1)^2}{\ln(1+x) - x}$$

3. (a) Find the equation of the tangent plane to the surface

$$\frac{x^2}{y} + \frac{y^2}{z} + \frac{z^2}{x} = 3$$

at the point $(1, 1, 1)$.

(b) Find the directional derivative of the function

$$F(x, y, z) = \frac{x^2}{y} + \frac{y^2}{z} + \frac{z^2}{x}$$

at the point $(1, 1, 1)$ in the direction $(-1, 2, 4)$.

4. (a) Reparametrize the curve

$$\mathbf{r}(t) = (2t, \cos t, \sin t)$$

in terms of arc length measured from the point where $t = 0$.

(b) For the curve in (a), find the unit tangent, unit principal normal and binormal vectors **T**,**N**,**B** of the Frenet-Serret formulas as well as the curvature at any point on the curve.

5. (a) Find and classify the critical points of

$$f(x,y) = x^2y - x^2 - y^2 - 2y$$

as local maxima, local minima or saddle points using the test involving the second partial derivatives of $f(x,y)$.

(b) Use the Lagrange multiplier method to find the shortest distance from the origin to the curve $xy^2 = 1$.

6. For each of the following double integrals

$$\text{(a)} \quad \int_0^1 \int_{x^{1/3}}^1 \sqrt{1-y^4}\, dy\, dx, \qquad \text{(b)} \iint_{x^2+y^2\le 1} \ln(x^2+y^2)\, dxdy,$$

sketch the domain of integration and evaluate the integral.

7. Find the volume of the region bounded by the cylinder $x^2 + y^2 = 2y$, the paraboloid $x^2 + y^2 = z$ and the plane $z = 0$.

8. Compute $\iiint_R xz\, dV$, where R is the solid tetrahedron with vertices

$$(0,0,0), (1,0,0), (1,1,0), (0,1,1).$$