McGill University April 2012
Faculty of Science Final examination

Principles of Statistics II

Math 204

Monday, April 23rd, 2012

Time: 6 pm – 9 pm

Examiner: Prof. Russell Steele Associate Examiner: Prof. David Stephens

INSTRUCTIONS

1. The seven questions have to be answered in the exam booklets provided
2. The total possible number of points for the exam is 180.
3. This is a closed book exam. One 8 1/2" × 11" double sided crib sheet is allowed.
4. Calculators (both programmable and non-programmable) are permitted.
5. Use of a regular dictionary is permitted.
6. Use of a translation dictionary is permitted.

This exam comprises the cover page, eight pages of questions and output, with questions numbered 1 to 7, and five pages of statistical tables.

1. (10 pts) A particular measure of ceramic strength was obtained for two different batches of ceramic material, with 10 random samples collected from each batch. The sample statistics for each batch are contained in the table below. Test the hypothesis that the *population* standard deviation for the first batch is larger than the *population* standard deviation for the second batch with Type I error rate $\alpha = 0.05$.

Batch	# of Samples	Mean	Standard deviation	Min.	Max.
1	10	671.08	71.68	518.65	751.67
2	10	610.4	56.06	531.37	747.54

2. (15 points) A study was designed to evaluate the effects of an herbal remedy, Echinacea purpurea, on upper respiratory infections (URI) in children. Children with URI, aged 2 to 11 years, were assigned to receive either echinacea or placebo (parents did not know the assignment) and then followed up after recovering from the illness. Parents were then asked to rate their child's severity of illness as mild, moderate, or severe. The results of the study are contained in the table below. Test the hypothesis that there is an association between the treatment variable and the parental assessment of severity. Use a Type I error rate of 0.10.

	Group	
Parental assessment	Echinacea	Placebo
Mild	153	170
Moderate	128	157
Severe	48	40

3. (25 pts) A certain suspect garage in the Plateau was suspected of insurance fraud by an insurance company. The insurance company took 10 damaged cars that had been serviced by the suspect garage to a more trusted garage and had a second damage estimate completed. Here are the damage estimates for the 10 automobiles at the two garages:

Car	Suspect Garage	Trusted Garage
1	1375	1250
2	1550	1300
3	1250	1250
4	1300	1200
5	900	950
6	1500	1575
7	1750	1600
8	3600	3300
9	2250	2125
10	2800	2600

(a) (10 points) Conduct a sign test to determine whether the suspect garage is charging higher estimates of damage at Type I error $\alpha = 0.05$).

(b) (10 points) Conduct a signed rank test to test the same hypothesis in part (a) (again at $\alpha = 0.05$). Do you come to the same conclusion?

(c) (5 points) Briefly state one reason why you may want to use one of the non-parametric tests in parts (a) or (b) instead of a paired t-test.

4. (30 pts) Three types (labelled A, B, and C) of soil preparation were each randomly installed in plots at four different locations (labelled 1, 2, 3 and 4), i.e. each type of preparation was installed at a random plot at each of the four locations. The researcher measured the growth of seedlings planted in each of the 12 plots and constructed the following ANOVA table (although some of the cells are missing):

	Df	Sum Sq	Mean Sq	F value
Soil Prep		48.667		
Location		51.333		
Residuals				
Total	11	108.667		

(a) (10 pts) Write down the ANOVA table above in your exam booklet, correctly filling in the missing cells.

(b) (5 pts) Using your answer to part (a), is there evidence to indicate mean differences in growth between the cell preparations at a significance level of $\alpha = 0.05$?

(c) (5 pts) Name the experimental design that was used.

(d) (5 pts) Construct the *one-way* ANOVA table that compares the three brands of soil treatment, *ignoring* the location factor.

(e) (5 pts) Using your answer to part (d), would you come to the same conclusion as in part (b)? Why or why not?

5. (30 pts) Some researchers believed that the iron content of food could be affected by the type of pot used to cook the food. The researchers conducted a study using three different kinds of Ethiopian cookware: iron, clay, and aluminum pots. They randomly selected 12 pots of each kind for the study (yielding 36 pots in total). They randomly assigned each pot to cook one of three different types of food (meat, legumes, or vegetables) in a completely randomized, balanced two factor design. The food was cooked for the same amount of time in each case and the iron content of the food was then measured.

(a) (5 points) List the different treatments for this experiment, identify the experimental unit and determine the number of experimental units assigned to each treatment for this design.

(b) (20 points) The two-way ANOVA table for the data and diagnostic plots for the model (Figure 1, next page) are below. Conduct a complete analysis of variance for the model below and clearly state your conclusions. Conduct all hypothesis tests at $\alpha = 0.01$. Be sure to state and assess validity of your assumptions for the model.

```
> iron.model = aov(iron~type*food)
> summary(iron.model)
            Df  Sum Sq Mean Sq F value    Pr(>F)
type         2 24.8940 12.4470  92.263 8.531e-13 ***
food         2  9.2969  4.6484  34.456 3.699e-08 ***
type:food    4  2.6404  0.6601   4.893  0.004247 **
Residuals   27  3.6425  0.1349
```

(c) (5 points) Could you conclude from the results in part (b) that a single type of pot would have, on average, higher iron levels for all three kinds of food? Why or why not?

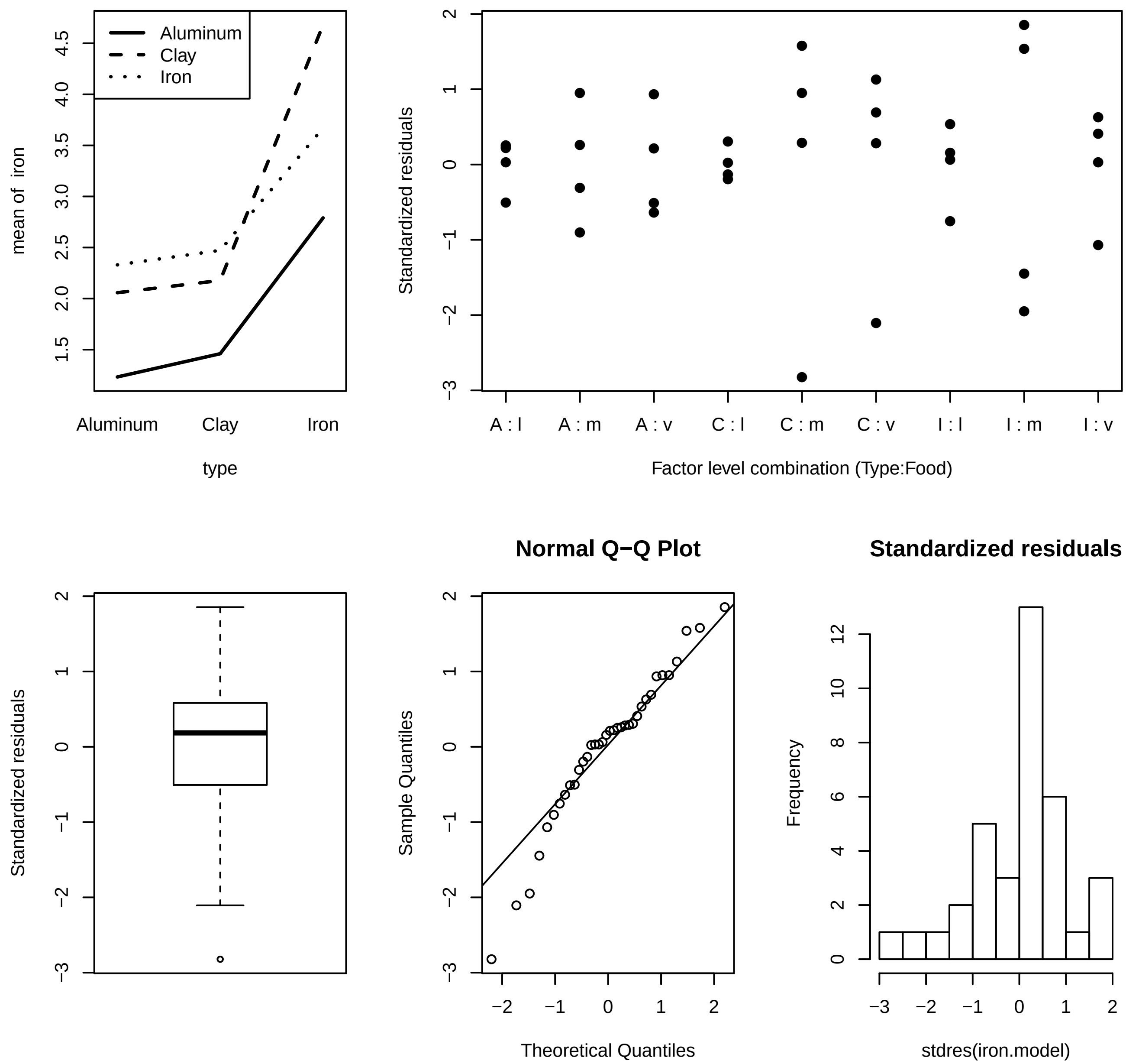


Figure 1: Diagnostic plots for Question 5

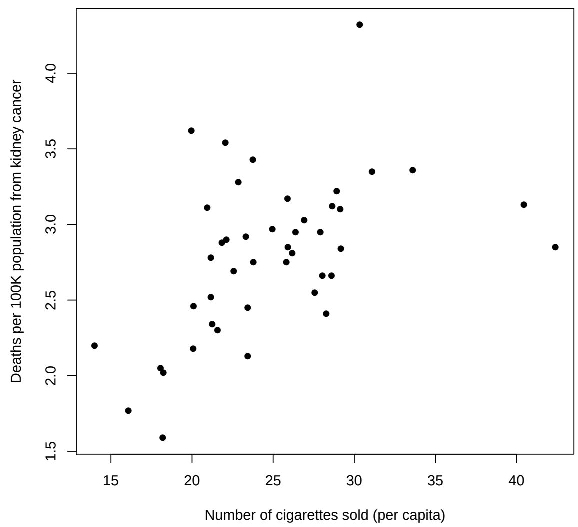


Figure 2: Plot of data for Question 6

6. (20 points) Fraumeni (1968, Journal of the National Cancer Institute) collected data from 43 states and the District of Columbia on the number of cigarettes sold per capita and deaths per 100K people from various kinds of cancer. The figure above (Figure 2) is a plot of the number of cigarettes sold and the number of deaths per 100K people from kidney cancer for each of the 44 observations. The regression model output for this data follows.

```
> kidney.model1 = lm(KID~CIG)
> summary(kidney.model1)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.66359    0.32020   5.196 5.63e-06 ***
CIG          0.04539    0.01255   3.617 0.000792 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.4586 on 42 degrees of freedom
Multiple R-squared: 0.2375,Adjusted R-squared: 0.2194
F-statistic: 13.09 on 1 and 42 DF,  p-value: 0.0007922
```

(a) (6 points) Test for a linear association between the number of cigarettes sold per capita and the number of kidney cancer deaths per 100K people with Type I error rate $\alpha = 0.05$.

(b) (5 points) What is the sample correlation between the number of deaths due to kidney cancer per 100K and the number of cigarettes sold per capita?

(c) (9 points) State the model assumptions that are necessary for your conclusions in part (b) to be valid. Assess the appropriateness of those assumptions using the figure above (Figure 2), the figure on the next page (Figure 3), and/or the output for the linear regression fit.

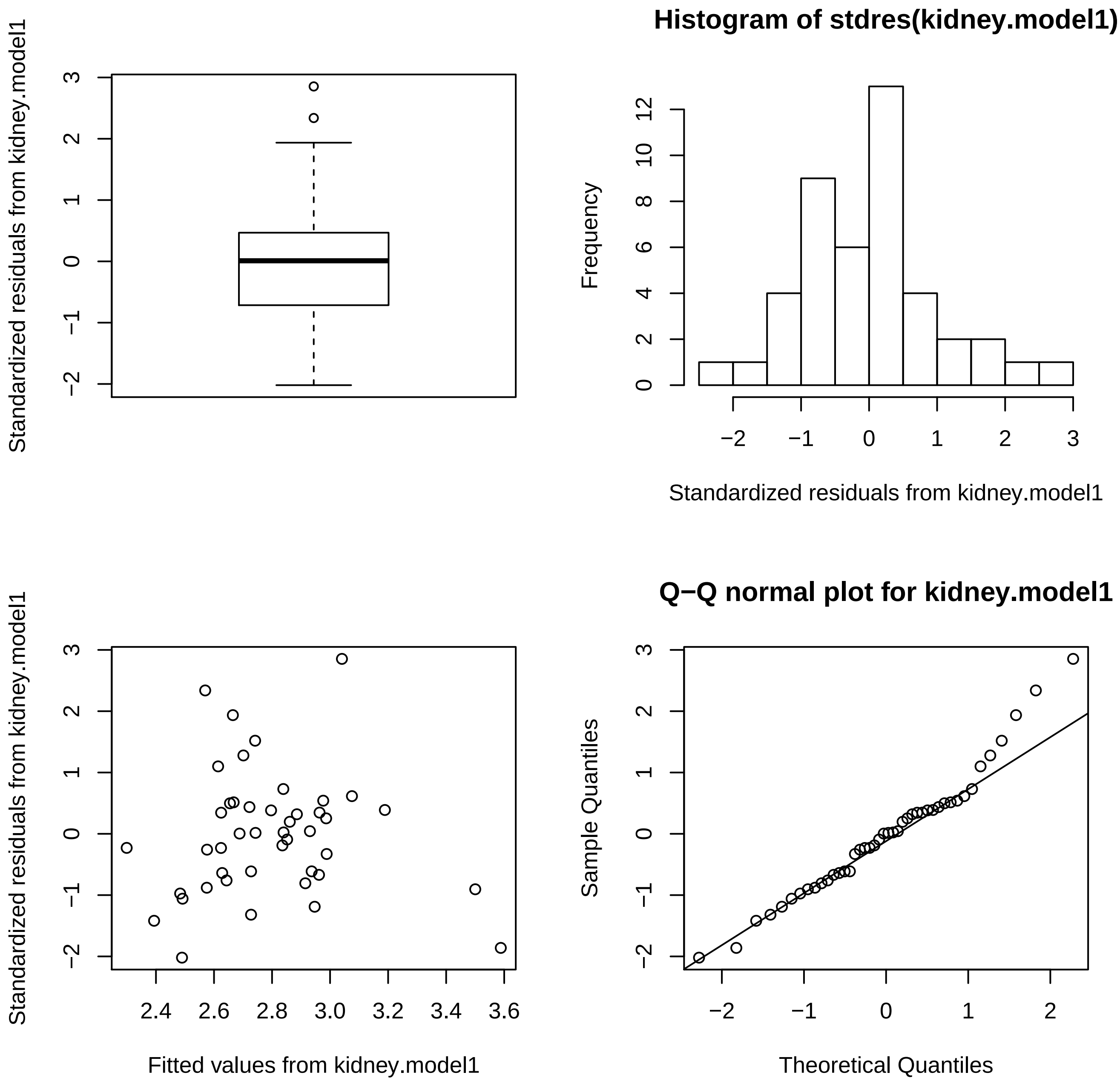


Figure 3: Diagnostic plots for Question 6

7. (50 points) It is assumed that wages will rise with experience (or length of service, LOS). A random sample of 60 women working in Indiana banks was taken. LOS is measured in months of experience and wages are yearly total income divided by number of weeks worked. The size of the bank where each woman worked was also measured and banks were classified into two different categories: Large and Small.

   Four different regression models were fit to the data. The regression output for these four models is contained on the next page. `los` indicates the LOS variable, `size` is the size of the bank.

   (a) (7 points) Using the output for `model1`, estimate the mean wage for a woman with LOS equal to 60 months. Provide an approximate 95% confidence interval for your estimate. *Hint: use the standard error of* $\hat{\beta}_1$ *OR the standard deviation of* `los` *to find* S_{XX}.

   (b) (4 points) Interpret the value of the two slope coefficients in `model3`.

   (c) (4 points) Using the output for `model4`, give a prediction for the wages for a woman with LOS of 60 months who is working at a large bank. **You do not need to provide a prediction interval.**

   (d) (5 points) Interpret the value of the interaction coefficient in `model4`.

   (e) (5 points) Using **only** the values of R^2 and adjusted R^2 for `model3` and `model4`, explain which of these two models should be preferred.

   (f) (8 points) Using the output for `model4`, test the hypothesis that the association between LOS and wages depends on the size of the bank with Type I error $\alpha = 0.01$. State your conclusion.

   (g) (6 points) Using *forward* step-wise regression and the output for all four models, choose an appropriate model for the data using F-tests and $\alpha = 0.05$.

   (h) (6 points) Using *backward* step-wise regression and the output for all four models, choose an appropriate model for the data using F-tests and $\alpha = 0.05$.

   (i) (5 points) Are your model selected in parts (g) and (h) the same model? Will this always be the case? Explain your answer.

```
> describe(los)
  var  n  mean    sd median trimmed   mad min max range skew kurtosis   se
1   1 60 70.48 51.73     60   62.69 44.48   7 228   221 1.33     1.42 6.68

> summary(size)
Large Small
   35    25
```

```
## Model 1
> summary(model1)

Call:
lm(formula = wages ~ los)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 44.21281    2.62824  16.822   <2e-16 ***
los          0.07310    0.03015   2.425   0.0185 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 11.98 on 58 degrees of freedom
Multiple R-squared: 0.09202,Adjusted R-squared: 0.07637
F-statistic: 5.878 on 1 and 58 DF,  p-value: 0.01847
> anova(model1)
Analysis of Variance Table

Response: wages
          Df Sum Sq Mean Sq F value  Pr(>F)
los        1  843.5  843.51  5.8782 0.01847 *
Residuals 58 8322.9  143.50

### Model 2

> model2 = lm(wages~size)
> summary(model2)

Call:
lm(formula = wages ~ size)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   53.216      1.975   26.94  < 2e-16 ***
sizeSmall     -9.242      3.060   -3.02  0.00375 **
---
Residual standard error: 11.69 on 58 degrees of freedom
Multiple R-squared: 0.1359,Adjusted R-squared: 0.121
F-statistic: 9.121 on 1 and 58 DF,  p-value: 0.003754

> anova(model2)
Analysis of Variance Table

Response: wages
          Df Sum Sq Mean Sq F value   Pr(>F)
size       1 1245.6 1245.60  9.1208 0.003754 **
Residuals 58 7920.8  136.57
---
```

```
> model3 = lm(wages~los + size)
> summary(model3)
Call:
lm(formula = wages ~ los + size)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  47.69407    2.59207  18.400  < 2e-16 ***
los           0.08417    0.02770   3.039 0.003582 **
sizeSmall   -10.22840    2.88197  -3.549 0.000782 ***
---
Residual standard error: 10.94 on 57 degrees of freedom
Multiple R-squared: 0.2564,Adjusted R-squared: 0.2303
F-statistic: 9.825 on 2 and 57 DF,  p-value: 0.0002157

> anova(model3)
Analysis of Variance Table

Response: wages
          Df Sum Sq Mean Sq F value    Pr(>F)
los        1  843.5  843.51  7.0535 0.0102409 *
size       1 1506.3 1506.35 12.5961 0.0007823 ***
Residuals 57 6816.6  119.59

# Model 4
> model4 = lm(wages~los*size)
> summary(model4)

Call:
lm(formula = wages ~ los * size)

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)    49.54532    3.37887  14.663  < 2e-16 ***
los             0.05595    0.04307   1.299  0.19925
sizeSmall     -13.63087    4.90998  -2.776  0.00747 **
los:sizeSmall   0.04828    0.05634   0.857  0.39511
---
Residual standard error: 10.96 on 56 degrees of freedom
Multiple R-squared: 0.266,Adjusted R-squared: 0.2267
F-statistic: 6.764 on 3 and 56 DF,  p-value: 0.0005667
> anova(model4)
Analysis of Variance Table

Response: wages
          Df Sum Sq Mean Sq F value    Pr(>F)
los        1  843.5  843.51  7.0206 0.0104534 *
size       1 1506.3 1506.35 12.5374 0.0008115 ***
los:size   1   88.2   88.24  0.7344 0.3951072
Residuals 56 6728.3  120.15
```

TABLE II Continued

d. $n = 8$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.923	.663	.430	.168	.058	.017	.004	.001	.000	.000	.000	.000	.000
1	.997	.943	.813	.503	.255	.106	.035	.009	.001	.000	.000	.000	.000
2	1.000	.994	.962	.797	.552	.315	.145	.050	.011	.001	.000	.000	.000
3	1.000	1.000	.995	.944	.806	.594	.363	.174	.058	.010	.000	.000	.000
4	1.000	1.000	1.000	.990	.942	.826	.637	.406	.194	.056	.005	.000	.000
5	1.000	1.000	1.000	.999	.989	.950	.855	.685	.448	.203	.038	.006	.000
6	1.000	1.000	1.000	1.000	.999	.991	.965	.894	.745	.497	.187	.057	.003
7	1.000	1.000	1.000	1.000	1.000	.999	.996	.983	.942	.832	.570	.337	.077

e. $n = 9$

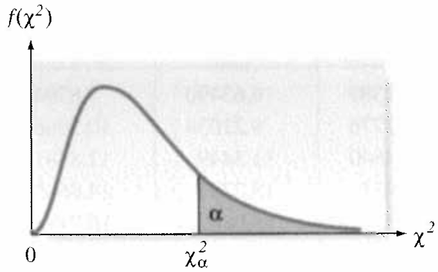
k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.914	.630	.387	.134	.040	.010	.002	.000	.000	.000	.000	.000	.000
1	.997	.929	.775	.436	.196	.071	.020	.004	.000	.000	.000	.000	.000
2	1.000	.992	.947	.738	.463	.232	.090	.025	.004	.000	.000	.000	.000
3	1.000	.999	.992	.914	.730	.483	.254	.099	.025	.003	.000	.000	.000
4	1.000	1.000	.999	.980	.901	.733	.500	.267	.099	.020	.001	.000	.000
5	1.000	1.000	1.000	.997	.975	.901	.746	.517	.270	.086	.008	.001	.000
6	1.000	1.000	1.000	1.000	.996	.975	.910	.768	.537	.262	.053	.008	.000
7	1.000	1.000	1.000	1.000	1.000	.996	.980	.929	.804	.564	.225	.071	.003
8	1.000	1.000	1.000	1.000	1.000	1.000	.998	.990	.960	.866	.613	.370	.086

f. $n = 10$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.904	.599	.349	.107	.028	.006	.001	.000	.000	.000	.000	.000	.000
1	.996	.914	.736	.376	.149	.046	.011	.002	.000	.000	.000	.000	.000
2	1.000	.988	.930	.678	.383	.167	.055	.012	.002	.000	.000	.000	.000
3	1.000	.999	.987	.879	.650	.382	.172	.055	.011	.001	.000	.000	.000
4	1.000	1.000	.998	.967	.850	.633	.377	.166	.047	.006	.000	.000	.000
5	1.000	1.000	1.000	.994	.953	.834	.623	.367	.150	.033	.002	.000	.000
6	1.000	1.000	1.000	.999	.989	.945	.828	.618	.350	.121	.013	.001	.000
7	1.000	1.000	1.000	1.000	.998	.988	.945	.833	.617	.322	.070	.012	.000
8	1.000	1.000	1.000	1.000	1.000	.998	.989	.954	.851	.624	.264	.086	.004
9	1.000	1.000	1.000	1.000	1.000	1.000	.999	.994	.972	.893	.651	.401	.096

(continued)

TABLE VII Critical Values of χ^2



Degrees of Freedom	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$
1	.0000393	.0001571	.0009821	.0039321	.0157908
2	.0100251	.0201007	.0506356	.102587	.210720
3	.0717212	.114832	.215795	.351846	.584375
4	.206990	.297110	.484419	.710721	1.063623
5	.411740	.554300	.831211	1.145476	1.61031
6	.675727	.872085	1.237347	1.63539	2.20413
7	.989265	1.239043	1.68987	2.16735	2.83311
8	1.344419	1.646482	2.17973	2.73264	3.48954
9	1.734926	2.087912	2.70039	3.32511	4.16816
10	2.15585	2.55821	3.24697	3.94030	4.86518
11	2.60321	3.05347	3.81575	4.57481	5.57779
12	3.07382	3.57056	4.40379	5.22603	6.30380
13	3.56503	4.10691	5.00874	5.89186	7.04150
14	4.07468	4.66043	5.62872	6.57063	7.78953
15	4.60094	5.22935	6.26214	7.26094	8.54675
16	5.14224	5.81221	6.90766	7.96164	9.31223
17	5.69724	6.40776	7.56418	8.67176	10.0852
18	6.26481	7.01491	8.23075	9.39046	10.8649
19	6.84398	7.63273	8.90655	10.1170	11.6509
20	7.43386	8.26040	9.59083	10.8508	12.4426
21	8.03366	8.89720	10.28293	11.5913	13.2396
22	8.64272	9.54249	10.9823	12.3380	14.0415
23	9.26042	10.19567	11.6885	13.0905	14.8479
24	9.88623	10.8564	12.4011	13.8484	15.6587
25	10.5197	11.5240	13.1197	14.6114	16.4734
26	11.1603	12.1981	13.8439	15.3791	17.2919
27	11.8076	12.8786	14.5733	16.1513	18.1138
28	12.4613	13.5648	15.3079	16.9279	18.9392
29	13.1211	14.2565	16.0471	17.7083	19.7677
30	13.7867	14.9535	16.7908	18.4926	20.5992
40	20.7065	22.1643	24.4331	26.5093	29.0505
50	27.9907	29.7067	32.3574	34.7642	37.6886
60	35.5346	37.4848	40.4817	43.1879	46.4589
70	43.2752	45.4418	48.7576	51.7393	55.3290
80	51.1720	53.5400	57.1532	60.3915	64.2778
90	59.1963	61.7541	65.6466	69.1260	73.2912
100	67.3276	70.0648	74.2219	77.9295	82.3581

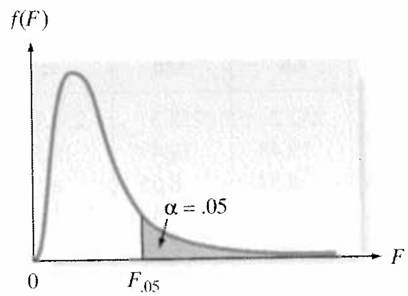
Source: From C. M. Thompson, "Tables of the Percentage Points of the χ^2-*Distribution*," *Biometrika*, 1941, 32, 188–189. Reproduced by permission of the *Biometrika* Trustees.

(continued)

TABLE VII Continued

Degrees of Freedom	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	2.70554	3.84146	5.02389	6.63490	7.87944
2	4.60517	5.99147	7.37776	9.21034	10.5966
3	6.25139	7.81473	9.34840	11.3449	12.8381
4	7.77944	9.48773	11.1433	13.2767	14.8602
5	9.23635	11.0705	12.8325	15.0863	16.7496
6	10.6446	12.5916	14.4494	16.8119	18.5476
7	12.0170	14.0671	16.0128	18.4753	20.2777
8	13.3616	15.5073	17.5346	20.0902	21.9550
9	14.6837	16.9190	19.0228	21.6660	23.5893
10	15.9871	18.3070	20.4831	23.2093	25.1882
11	17.2750	19.6751	21.9200	24.7250	26.7569
12	18.5494	21.0261	23.3367	26.2170	28.2995
13	19.8119	22.3621	24.7356	27.6883	29.8194
14	21.0642	23.6848	26.1190	29.1413	31.3193
15	22.3072	24.9958	27.4884	30.5779	32.8013
16	23.5418	26.2962	28.8454	31.9999	34.2672
17	24.7690	27.5871	30.1910	33.4087	35.7185
18	25.9894	28.8693	31.5264	34.8053	37.1564
19	27.2036	30.1435	32.8523	36.1908	38.5822
20	28.4120	31.4104	34.1696	37.5662	39.9968
21	29.6151	32.6705	35.4789	38.9321	41.4010
22	30.8133	33.9244	36.7807	40.2894	42.7956
23	32.0069	35.1725	38.0757	41.6384	44.1813
24	33.1963	36.4151	39.3641	42.9798	45.5585
25	34.3816	37.6525	40.6465	44.3141	46.9278
26	35.5631	38.8852	41.9232	45.6417	48.2899
27	36.7412	40.1133	43.1944	46.9630	49.6449
28	37.9159	41.3372	44.4607	48.2782	50.9933
29	39.0875	42.5569	45.7222	49.5879	52.3356
30	40.2560	43.7729	46.9792	50.8922	53.6720
40	51.8050	55.7585	59.3417	63.6907	66.7659
50	63.1671	67.5048	71.4202	76.1539	79.4900
60	74.3970	79.0819	83.2976	88.3794	91.9517
70	85.5271	90.5312	95.0231	100.425	104.215
80	96.5782	101.879	106.629	112.329	116.321
90	107.565	113.145	118.136	124.116	128.299
100	118.498	124.342	129.561	135.807	140.169

TABLE IX Percentage Points of the F-distribution, $\alpha = .05$



ν_2 \ ν_1 (DENOMINATOR DEGREES OF FREEDOM \ NUMERATOR DEGREES OF FREEDOM)	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.77
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

Source: From M. Merrington and C. M. Thompson, "Tables of Percentage Points of the Inverted Beta (F)-Distribution," *Biometrika*, 1943, 33, 73–88.

(continued)

TABLE XIII Critical Values of T_0 in the Wilcoxon Paired Difference Signed Rank Test

One-Tailed	Two-Tailed	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$
$\alpha = .05$	$\alpha = .10$	1	2	4	6	8	11
$\alpha = .025$	$\alpha = .05$		1	2	4	6	8
$\alpha = .01$	$\alpha = .02$			0	2	3	5
$\alpha = .005$	$\alpha = .01$				0	2	3
		$n = 11$	$n = 12$	$n = 13$	$n = 14$	$n = 15$	$n = 16$
$\alpha = .05$	$\alpha = .10$	14	17	21	26	30	36
$\alpha = .025$	$\alpha = .05$	11	14	17	21	25	30
$\alpha = .01$	$\alpha = .02$	7	10	13	16	20	24
$\alpha = .005$	$\alpha = .01$	5	7	10	13	16	19
		$n = 17$	$n = 18$	$n = 19$	$n = 20$	$n = 21$	$n = 22$
$\alpha = .05$	$\alpha = .10$	41	47	54	60	68	75
$\alpha = .025$	$\alpha = .05$	35	40	46	52	59	66
$\alpha = .01$	$\alpha = .02$	28	33	38	43	49	56
$\alpha = .005$	$\alpha = .01$	23	28	32	37	43	49
		$n = 23$	$n = 24$	$n = 25$	$n = 26$	$n = 27$	$n = 28$
$\alpha = .05$	$\alpha = .10$	83	92	101	110	120	130
$\alpha = .025$	$\alpha = .05$	73	81	90	98	107	117
$\alpha = .01$	$\alpha = .02$	62	69	77	85	93	102
$\alpha = .005$	$\alpha = .01$	55	61	68	76	84	92
		$n = 29$	$n = 30$	$n = 31$	$n = 32$	$n = 33$	$n = 34$
$\alpha = .05$	$\alpha = .10$	141	152	163	175	188	201
$\alpha = .025$	$\alpha = .05$	127	137	148	159	171	183
$\alpha = .01$	$\alpha = .02$	111	120	130	141	151	162
$\alpha = .005$	$\alpha = .01$	100	109	118	128	138	149
		$n = 35$	$n = 36$	$n = 37$	$n = 38$	$n = 39$	
$\alpha = .05$	$\alpha = .10$	214	228	242	256	271	
$\alpha = .025$	$\alpha = .05$	195	208	222	235	250	
$\alpha = .01$	$\alpha = .02$	174	186	198	211	224	
$\alpha = .005$	$\alpha = .01$	160	171	183	195	208	
		$n = 40$	$n = 41$	$n = 42$	$n = 43$	$n = 44$	$n = 45$
$\alpha = .05$	$\alpha = .10$	287	303	319	336	353	371
$\alpha = .025$	$\alpha = .05$	264	279	295	311	327	344
$\alpha = .01$	$\alpha = .02$	238	252	267	281	297	313
$\alpha = .005$	$\alpha = .01$	221	234	248	262	277	292
		$n = 46$	$n = 47$	$n = 48$	$n = 49$	$n = 50$	
$\alpha = .05$	$\alpha = .10$	389	408	427	446	466	
$\alpha = .025$	$\alpha = .05$	361	379	397	415	434	
$\alpha = .01$	$\alpha = .02$	329	345	362	380	398	
$\alpha = .005$	$\alpha = .01$	307	323	339	356	373	

Source: From F. Wilcoxon and R. A. Wilcox, "Some Rapid Approximate Statistical Procedures," 1964, p. 28.