1. (20 marks) Evaluate the following integrals

(a) $\displaystyle\int_0^1 \arctan x \, dx$ (b) $\displaystyle\int_1^2 \frac{2x^2+3}{x(x+1)^2} dx$

2. (i) (10 marks) Find the length of the catenary $y = \cosh x$ from $x = -a$ to $x = a$.
   (ii) (10 marks) Find the approximation to the integral

$$\int_{-2}^{2} \frac{1}{1+x^2} dx$$

using the Simpson's approximation and four intervals (i.e. five function evaluations).

3. (i) (10 marks) Use Green's Theorem to evaluate the line integral

$$\int_C (1 + 10xy + y^2)dx + (6xy + 5x^2)dy$$

where C denotes the boundary of the square $0 \le x \le 1$, $0 \le y \le 1$ traversed in the anticlockwise sense.
   (ii) (10 marks) Find the surface area of the portion of the surface $3z = x^{\frac{3}{2}} + y^{\frac{3}{2}}$ that lies above the square $0 \le x \le 1$, $0 \le y \le 1$.

4. (i) (10 marks) Determine the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} x^{2n}$$

   (ii) (10 marks) Use the Binomial Theorem to expand the function $(1 - 4x^2)^{-\frac{1}{2}}$ as a series in powers of x.

5. (i) (10 marks) Sketch the curve given in polar coordinates by $r = \cos 2\theta$ as θ runs from 0 to 2π.
(ii) (10 marks) Find the total area enclosed by the curve you have sketched in (i).

6. (i) (7 marks) Find an equation for the circle $(x-1)^2 + y^2 = 1$ in polar coordinates. Give a range of θ for which the circle is traversed once.
(ii) (13 marks) Using cylindrical coordinates and the equation you have found in (i), find the volume of the region of 3-space given by the inequalities

$$(x-1)^2 + y^2 \leq 1, \quad z \geq 0 \quad \text{and} \quad z^2 \leq x^2 + y^2$$

7. (i) (10 marks) Find the value of the iterated integral

$$\int_{y=0}^{\frac{\pi}{2}} \left\{ \int_{x=y}^{\frac{\pi}{2}} \frac{\sin x}{x} dx \right\} dy.$$

Hint: You cannot compute the inner integral directly in terms of transcendental functions.
(ii) (10 marks) Using spherical coordinates, determine the volume common to a sphere of radius a and a cone with apex at the centre of the sphere and opening half-angle α. For example, the region of 3-space given by

$$x^2 + y^2 + z^2 \leq a^2, \quad z \geq (x^2 + y^2)^{\frac{1}{2}} \cot \alpha.$$

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FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-151B

Calculus B

Examiner: Professor S. W. Drury
Associate Examiner: Professor W. G. Brown

Date: Friday, 16 April 1999
Time: 9: 00 am. – 12: 00 noon

INSTRUCTIONS

All seven questions should be attempted for full credit.

This is a closed book examination.
Write your answers in the booklets provided.
All questions are of equal weight; each is worth 20 marks.
No calculators are allowed.

This exam comprises the cover and 2 pages of questions.