VERSION **1**

McGill University — Faculty of Science

FINAL EXAMINATION

MATHEMATICS 141 2012 01 CALCULUS 2

EXAMINER: Professor S. W. Drury DATE: Tuesday April 17, 2012
ASSOCIATE EXAMINER: Professor A. Hundemer TIME: 2 pm. to 5 pm.

FAMILY NAME:

GIVEN NAMES:

MR, MISS, MS, MRS, &c.: STUDENT NUMBER:

If you expect to graduate in Spring 2012 put an $\times$ in this box: ☐

INSTRUCTIONS

1. Fill in the above clearly. Enter your name as it appears on your student card.
2. Do not tear pages from this book; all your writing — even rough work — must be handed in. You may do rough work anywhere in the booklet except in the box below and in the answer boxes.
3. This is a closed book examination. Calculators are not permitted, but regular and translation dictionaries are permitted.
4. The questions are of two types:
   - In BRIEF SOLUTIONS questions, each answer will be marked right or wrong.
   - In SHOW ALL YOUR WORK questions a correct answer alone will not be sufficient unless substantiated by your work. Partial marks may be awarded for a partly correct answer.

   Begin your solution on the page where the question is printed. You may continue a solution *on the facing page*, or on the continuation pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed! Write your final answer in the answer box if provided. Within a question, each answer has equal weight. You are expected to simplify all answers wherever possible unless specifically instructed otherwise.
5. This examination booklet consists of this cover, Pages 1–7 containing 7 questions and Pages 8–11 which are blank continuation pages.
6. A TOTAL OF 84 POINTS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

1 /12	2 /12	3 /12		code
4 /12	5 /12	6 /12	7 /12	total /84

Note: This document has been compressed for ease of printing. On the actual exam, each question took up an entire page.

1. (12 points) $\boxed{\text{BRIEF SOLUTIONS}}$

(i) Find the Riemann sum for $\int_{-2}^{4} |x^3| dx$ which uses three intervals of equal length and evaluation points at the midpoints of these intervals.

ANSWER ONLY

(ii) Find $\int_{-2}^{4} |x^3| dx$.

ANSWER ONLY

(iii) Find $\lim_{n \to \infty} \frac{1}{n^8} \sum_{k=1}^{n} k^7$.

ANSWER ONLY

(iv) If $\frac{x^4}{(x+1)(x^2+5)} = Ax + B + \frac{C}{x+1} + \frac{Dx+E}{x^2+5}$, find C.

ANSWER ONLY

2. (12 points) $\boxed{\text{BRIEF SOLUTIONS}}$

(i) Write down a definite integral with respect to the variable y representing the area in the xy-plane enclosed by the curves $x = 6y$ and $x = y^2 + 5$. Do not evaluate the integral.

ANSWER ONLY

(ii) Find $\dfrac{d}{dx}\displaystyle\int_{1-x^2}^{1+x^2} \ln\left(1+\sqrt{t}\right)dt$ for $|x| < 1$.

ANSWER ONLY

(iii) Find the volume of the solid obtained by rotating the region $0 \leq y \leq x(1-x)$ of the xy-plane about the y-axis.

ANSWER ONLY

3. (12 points) $\boxed{\text{BRIEF SOLUTIONS}}$

(i) Find all values of s such that the tangent to the parametric curve $x = t^2$, $y = 2t - t^3$ at the point given by $t = s$ passes through the point $(x, y) = (3, 0)$.

ANSWER ONLY

(ii) Find the area enclosed by the polar loop $r = \sqrt{3 + 2\sin(\theta)}$.

ANSWER ONLY

(iii) Find the arclength of the section of the polar curve $r = e^{\theta}$ given by $-\pi \leq \theta \leq \pi$.

ANSWER ONLY

4. (12 points) $\boxed{\text{SHOW ALL YOUR WORK!}}$

(i) Find $\displaystyle\int_0^1 x^2 \ln(x)dx.$

ANSWER ONLY

(ii) Find $\displaystyle\int_0^{\frac{\pi}{4}} \frac{(\sin(x))^2}{\cos(x)}dx.$

ANSWER ONLY

(iii) Find $\displaystyle\int_0^4 \frac{x}{\sqrt{x^2+2x+25}}dx.$

ANSWER ONLY

5. (12 points) $\boxed{\text{SHOW ALL YOUR WORK!}}$ The piece of the parametric curve $x = \sqrt{t}(3-t)$, $y = 3t$ corresponding to $1 \le t \le 2$ is rotated about the y-axis. Find the surface area of the resulting surface.

6. (12 points) $\boxed{\text{SHOW ALL YOUR WORK!}}$ Find the volume of the solid obtained by rotating the region $0 \le y \le x\Big(1+\cos(x)\Big)$, $0 \le x \le \pi$ of the xy-plane about the x-axis.

7. (12 points) $\boxed{\text{SHOW ALL YOUR WORK!}}$ For each of the following series you should apply one or more tests to determine whether the series is absolutely convergent, conditionally convergent or divergent. All tests used must be named and all statements carefully justified.

(i) $\displaystyle\sum_{n=1}^{\infty} n \arctan\left(\frac{1}{n^2}\right)$

ANSWER ONLY

(ii) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(n+2)^n}{n^{n+2}}$

ANSWER ONLY