VERSION **1**

McGILL UNIVERSITY
FACULTY OF SCIENCE
FINAL EXAMINATION

MATHEMATICS 141 2006 01 — CALCULUS II

EXAMINER: Professor W. G. Brown
ASSOCIATE EXAMINER: Prof. N. Sancho

DATE: Wednesday, April 12th, 2006
TIME: 14:00 – 17:00 hours

FAMILY NAME:
GIVEN NAMES:

STUDENT NUMBER:

INSTRUCTIONS

1. Fill in the above clearly.
2. Do not tear pages from this book; all your writing — even rough work — must be handed in. You may do rough work for this paper anywhere in the booklet.
3. Calculators are not permitted. CLOSED BOOK EXAM. REGULAR + TRANSLATION DICTIONARIES ALLOWED
4. This examination booklet consists of this cover, Pages 1 through 8 containing questions; and Pages 9, 10, and 11, which are blank. *Your neighbour's version of this test may be different from yours.*
5. There are two kinds of problems on this examination, each clearly marked as to its type.
   - Most of the questions on this paper require that you SHOW ALL YOUR WORK! Their solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
   - Some of the questions on this paper require only BRIEF SOLUTIONS ; for these you are expected to write the correct answer in the box provided; you are not asked to show your work, and you should not expect partial marks for solutions that are not correct.

   You are expected to simplify your answers wherever possible.

   You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)
6. A TOTAL OF 100 MARKS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a) /4	1(b) /3	1(c) /3	1(d) /4	2(a) /4	2(b) /4	2(c) /4	3(a) /3	3(b) /3
3(c) /3	3(d) /3		4 /12	5(a) /4	5(b) /5	5(c) /4	6(a) /8	6(b) /4
7(a) /5	7(b) /5	7(c) /3	8 /12					TOTAL /100

1. SHOW ALL YOUR WORK!

   (a) [4 MARKS] Evaluate $\int_{-1}^{2} |x|\, dx$.

   (b) [3 MARKS] Evaluate $\int_{1}^{e^3} \frac{dt}{t\sqrt{1+\ln t}}$.

   (c) [3 MARKS] Evaluate $\frac{d}{dx}\int_{0}^{x^2} e^{t^2}\, dt$.

   (d) [4 MARKS] Evaluate

$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{0}{n}\right)^7 + \left(\frac{1}{n}\right)^7 + \left(\frac{2}{n}\right)^7 + \ldots + \left(\frac{n-1}{n}\right)^7\right].$$

SHOW ALL YOUR WORK!

2. For each of the following series you are expected to apply one or more tests for convergence or divergence and determine whether the series is convergent. In each case you must answer 3 questions:

   - Name the test(s) that you are using.
   - Explain why the test(s) you have chosen is/are applicable to the given series.
   - Use the test(s) to conclude whether or not the series is convergent.

   (a) [4 MARKS] $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\tanh n)^2 + 1}$

   (b) [4 MARKS] $\displaystyle\sum_{n=1}^{\infty} n^{2n} e^{-n^2}$

   (c) [4 MARKS] $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 - 85n + 12}{n(n+6)^2}$

3. BRIEF SOLUTIONS Express the value of each of the following as a definite integral or a sum, product, or quotient of several definite integrals, but *do not evaluate the integral(s)*. It is not enough to quote a general formula: your integrals must have integrand and limits specific to the given problems, and should be simplified as much as possible, except that you are not expected to evaluate the integrals.

   (a) [3 MARKS] Expressed as integral(s) along the x-axis only, the area of the region bounded by the parabola $y^2 = 2x + 6$ and the line $y = x - 1$. An answer involving integration along the y-axis will not be accepted.

   DEFINITE INTEGRAL(S) ONLY (DO NOT EVALUATE)

   (b) [3 MARKS] The volume of the solid obtained by rotating about the line $y = 1$ the region bounded by the curves $y = x^3$ and $y = x^2$. For this question you are to use only the method of "washers".

   DEFINITE INTEGRAL(S) ONLY (DO NOT EVALUATE)

   (c) [3 MARKS] The volume of the solid obtained by rotating about the line $y = 1$ the region bounded by the curves $y = x^3$ and $y = x^2$. For this question you are to use only the method of "cylindrical shells".

   DEFINITE INTEGRAL(S) ONLY (DO NOT EVALUATE)

   (d) [3 MARKS] The length of the curve whose equation is

$$\frac{x^2}{4} + \frac{y^2}{9} = 1\,.$$

   DEFINITE INTEGRAL(S) ONLY (DO NOT EVALUATE)

4. SHOW ALL YOUR WORK!

[12 MARKS] Evaluate the indefinite integral

$$\int \frac{x^5 + x}{x^4 - 16}\, dx\,.$$

5. SHOW ALL YOUR WORK!

Showing all your work, evaluate each of the following:

(a) [4 MARKS] $\int \cos x \cdot \cosh x \, dx$

(b) [5 MARKS] $\int_{-3}^{1} \sqrt{x^2 + 2x + 5} \, dx$

(c) [4 MARKS] $\int \sin^2 x \cdot \cos^2 x \, dx$

6. SHOW ALL YOUR WORK!

Consider the curve C defined by

$$\begin{aligned} x = x(t) &= 10 - 3t^2 \\ y = y(t) &= t^3 - 3t\,, \end{aligned}$$

where $-\infty < t < +\infty$.

(a) [8 MARKS] Determine the value of $\dfrac{d^2y}{dx^2}$ at the points where the tangent is horizontal.

(b) [4 MARKS] Determine the area of the surface of revolution about the x-axis of the arc

$$\left\{(x(t), y(t)) : -\sqrt{3} \leq t \leq 0\right\}.$$

7. SHOW ALL YOUR WORK!

(a) [5 MARKS] Showing detailed work, determine whether the following integral is convergent; if it is convergent, determine its value:

$$\int_{-1}^{0} \frac{dx}{x^{\frac{2}{3}}}.$$

(b) [5 MARKS] Determine whether the following series is conditionally convergent, absolutely convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n - \ln n}$$

(c) [3 MARKS] Give an example of a sequence $\{a_n\}$ with the property that $\lim_{n\to\infty} a_n = 0$ but $\sum_{n=1}^{\infty} a_n = +\infty$. You are expected to give a formula for the general term a_n of your sequence.

8. SHOW ALL YOUR WORK!

[12 MARKS] The arc

$$r = 1 - \cos\theta \qquad (0 \leq \theta \leq \pi)$$

divides the area bounded by the curve

$$r = 1 + \sin\theta \qquad (0 \leq \theta \leq 2\pi)$$

into two parts. Showing all your work, carefully find the area of the part that contains the point $(r, \theta) = \left(\frac{1}{2}, \frac{\pi}{2}\right)$.

CONTINUATION PAGE FOR PROBLEM NUMBER

You *must* refer to this continuation page on the page where the problem is printed!

CONTINUATION PAGE FOR PROBLEM NUMBER

You *must* refer to this continuation page on the page where the problem is printed!

CONTINUATION PAGE FOR PROBLEM NUMBER

You *must* refer to this continuation page on the page where the problem is printed!