McGill University

Faculty of Science

FINAL EXAMINATION

Math 141: Calculus 2

Examiner: Dr. Aaron Lauve
Associate Examiner: Dr. William Brown

Date: Monday, December 11, 2006
Time: 9:00 AM–12:00 PM

Name: ____________________	Student ID: ____________

Instructions

1. Check that this exam-booklet comprises: this cover page, **nine(9) pages of questions,** and 4 additional (blank) pages.
2. Answer each question in the space provided for that question.
3. If your answer does not fit on the front and back of a question page, use one of the blank pages provided at the end of this booklet.
4. Calculators are not permitted.
5. This is a closed book, no notes exam.
6. Partial credit may be awarded if you show all your work.
7. Use of a regular and translation dictionaries are permitted.
8. Please do not take apart this exam-booklet as it must be returned.

Good Luck et Bonne Chance!!

Mark Distribution & Tally

Question	1	2	3	4	5	6	7	8	9	**Tally**
Possible	5	8	8	15	20	16	8	10	10	100
Score										

1. Consider the function $y = f(x) = |3 - x|$.

   (a) **(2 marks)** Compute the average value of f on the interval $[0, 5]$.

   (b) **(3 marks)** Make a sketch of the graph of $F(x) = \int_0^x f(t)\,dt$.

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2. (a) **(3 marks)** If $f(x) = \dfrac{e^{\sqrt{x}+1}}{\sqrt{x}}$, find the area under the graph of f on the interval $4 \le x \le 9$;

(b) **(2 marks)** If $g(t) = e^{t^2+1}$, compute $\dfrac{d}{dx}\displaystyle\int_{-1}^{x} g(t)\,dt$;

(c) **(3 marks)** If $g(t) = e^{t^2+1}$, compute $\dfrac{d}{dx}\displaystyle\int_{-x}^{\sqrt{x}} g(t)\,dt$.

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3. For each of the following series, prove it diverges or prove it converges and give the sum. You may proceed by analyzing partial sums or by applying any relevant series tests. For full credit, you must evaluate any limits involved carefully and completely.

   (a) **(4 marks)** $\sum_{n=1}^{\infty} \left\{ \ln\left(\frac{n}{n+1}\right) - \ln\left(\frac{n+2}{n+3}\right) \right\}$

   (b) **(2 marks)** $\sum_{n=9}^{\infty} e^{-n}(-1)^n$

   (c) **(2 marks)** $\sum_{n=1}^{\infty} \left\{ 4^n (2 + \sin n) - \frac{4^n}{2 + \sin n} \right\}$.

4. (a) **(10 marks)** Starting with the substitution $u = x^{1/2}$, compute $\displaystyle\int \frac{dx}{x^2 + x^{1/2}}$.

(b) **(5 marks)** Consider the curve $f(x) = \begin{cases} \frac{1}{x^{1/2}} & \text{for } 0 < x \leq 1 \\ \frac{1}{x^2} & \text{for } 1 < x < \infty \end{cases}$

Let $\mathcal{R}$ be the region bounded by $x = 0$, $y = 0$, and f. Find the area of $\mathcal{R}$.

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5. Let $\mathcal{R}$ be the region in the first quadrant bounded by $x = 0$, $y = \pi/3$, and $y = \arctan x$. Let $\mathcal{C}$ be the segment of $y = \arctan x$ bounding $\mathcal{R}$.

   (a) For each of the geometric quantities below, **set up** (but do not evaluate) an integral which would compute the requested value.

      i. **(2 marks)** The volume obtained by rotating $\mathcal{R}$ about the y-axis.

      ii. **(2 marks)** The volume obtained by rotating $\mathcal{R}$ about the line $y = \pi/3$.

      iii. **(2 marks)** The surface area obtained by rotating $\mathcal{C}$ about the y-axis.

      iv. **(2 marks)** The surface area obtained by rotating $\mathcal{C}$ about the line $y = \pi/3$.

   (b) **(12 marks)** Compute **two** of the quantities above (You choose which two).

6. **(16 marks)** Determine the convergence or divergence of each series below using the Alternating Series, Integral, Ratio, or Comparison Tests. For full credit, you must verify/show that the test you use on a given series is applicable.

(a) $\displaystyle\sum_{n=0}^{\infty} ne^{-n}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1+\cos n}{n^6+n^2}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1+2(-1)^n}{3^{2n}}$ (d) $\displaystyle\sum_{n=1}^{\infty} e^{-n}n!$

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7. Consider the parametric curve $\mathcal{C}$ described by $\begin{cases} x = t^3 - 3t \\ y = \sqrt{4-t^2} \end{cases}$ $(-\sqrt{3} \leq t \leq \sqrt{3})$.

(a) **(4 marks)** Find the locations (coordinates) of all vertical and horizontal tangent lines to $\mathcal{C}$ on $-\sqrt{3} < t < \sqrt{3}$.

(b) **(2 marks)** Verify that the curve $\mathcal{C}$ does not have a unique tangent line at $(x, y) = (0, 1)$.

(c) **(2 marks)** Finally, **set up** (but do not evaluate) an integral computing the length of the segment of $\mathcal{C}$ traced out during $-1 \leq t \leq 1$.

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8. The polar curve $r = \sin 2\theta$ restricted to the first quadrant describes a region $\mathcal{R}$ in the plane. The polar curve $r = \cos\theta$ cuts $\mathcal{R}$ into two (unequal) parts.

   (a) **(2 marks)** Sketch $\mathcal{R}$ and the curve that cuts it into two parts.

   (b) **(8 marks)** Find the area of the bottom part of $\mathcal{R}$.

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9. Compute the indicated integrals:

(a) **(3 marks)** $\lim_{n\to\infty} \frac{e-1}{n} \sum_{i=1}^{n} \left(1 + \frac{(e-1)i}{n}\right)^{-1}$;

(b) **(3 marks)** $\int_0^1 \frac{(4-x^2)+1}{(4-x^2)^{3/2}}\, dx$;

(c) **(4 marks)** $\int \frac{\sin x}{\sin x - \cos x}\, dx$.

FILL IN THE BLANK (ONE QUESTION PER PAGE):

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Fill in the blank (one question per page):

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