

1. **SHOW ALL YOUR WORK!**

(a) [5 MARKS] Evaluate $\int_1^2 \frac{2x+1}{2x^2+x+1} dx$.

(b) [5 MARKS] Evaluate $\int x^2 \sin x dx$.

(c) [5 MARKS] Evaluate $\int \frac{\cos x}{\sqrt{\sin x}} dx$.

(d) [5 MARKS] Evaluate $\frac{d}{dx} \int_{x+1}^{x+3} e^{\sqrt{t}} dt$.

SHOW ALL YOUR WORK!

2. For each of the following series you are expected to apply one or more tests for convergence or divergence and determine whether the series is convergent. In each case you must answer 3 questions:

- Name the test(s) that you are using.
- Explain why the test(s) you have chosen is/are applicable to the given series.
- Use the test(s) to conclude whether or not the series is convergent.

(a) [5 MARKS] $\sum_{n=3}^{\infty} \frac{1}{n^2 \ln n}$

(b) [5 MARKS] $\sum_{n=2}^{\infty} 2^{1/(n^2-n)}$

(c) [5 MARKS] $\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$

3. **SHOW ALL YOUR WORK!**

[10 MARKS] The region bounded by the x-axis and the curve

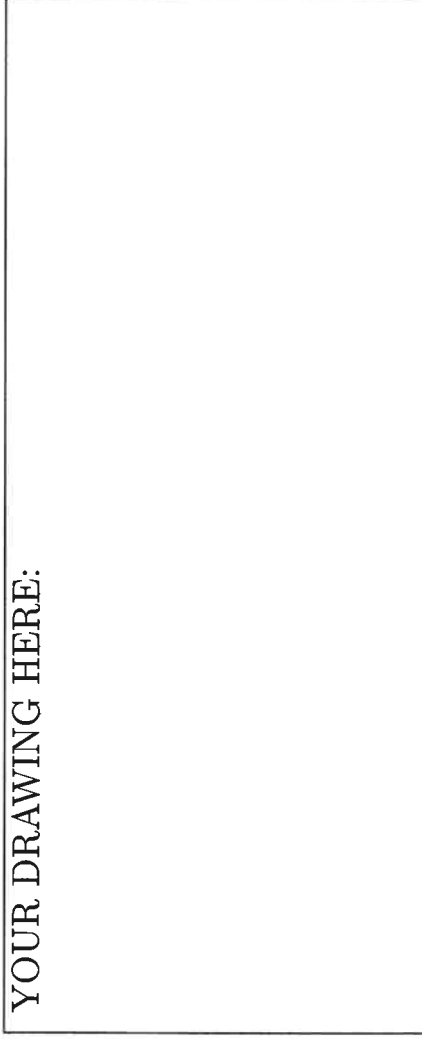
$$y = \sin(4x^2)$$

for $0 \leq x \leq \frac{1}{2}\sqrt{\pi}$ is rotated about the y-axis. Find the volume of the resulting solid.

4. **BRIEF SOLUTIONS** In part (b) of this question you are not requested to evaluate the resulting integral, but just to express your answer as a definite integral. It is not enough to quote a general formula: your integral must have integrand and limits specific to the given problem.

(a) [5 MARKS] Draw the graph of the curve given in polar coordinates by $r = \frac{\pi}{\theta}$, for $\pi/2 \leq \theta \leq 6\pi$.

YOUR DRAWING HERE:



(b) [5 MARKS] Find the length of the above curve

DEFINITE INTEGRAL ONLY (DO NOT EVALUATE)



5. **SHOW ALL YOUR WORK!**

Check the convergence of the following integrals. Whenever the integral converges, evaluate it.

- [5 MARKS] $\int_e^\infty \frac{dx}{x(\ln x)^2}$
- [5 MARKS] $\int_0^{\pi/2} \frac{dx}{\cos^2 x}$

6. **SHOW ALL YOUR WORK!**

(a) [6 MARKS] Evaluate: $\lim_{n \rightarrow \infty} \frac{(n!)^2}{(2n)!}$

(b) [6 MARKS] Evaluate: $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \sin(\pi i/n)$. Hint: use Riemann sums.

7. **SHOW ALL YOUR WORK!**

Consider the curve C defined by

$$\begin{aligned}x &= e^t - t \\y &= e^t + t.\end{aligned}$$

- (a) [6 MARKS] Find the tangent lines to C at $(1, 1)$ and at $(e - 1, e + 1)$.
(b) [6 MARKS] Find the area of the region between the curve C and the x-axis for $1 \leq t \leq 3$.

8. **SHOW ALL YOUR WORK!**

- (a) [5 MARKS] Find the sum

$$\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+7)}$$

Hint: Find a partial fractions decomposition, then expand and cancel out the expression for the n th partial sum.

- (b) [6 MARKS] Given the series

$$\sum_{n=1}^{\infty} \frac{1}{(n + \frac{1}{2})^{1.1}} = S,$$

use the integral test to find an upper bound on the error term $S - S_n$.

CONTINUATION PAGE FOR PROBLEM NUMBER

You *must* refer to this continuation page on the page where the problem is printed!

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