McGILL UNIVERSITY
FACULTY OF SCIENCE
FINAL EXAMINATION

MATH 141 - CALCULUS II - FALL TERM 2005

EXAMINER: Dr. G. Alon
ASSOCIATE EXAMINER: Prof. N. Sancho

DATE: Friday, December 9th 2005

FAMILY NAME:
GIVEN NAMES:

STUDENT NUMBER:

INSTRUCTIONS

1. Fill in the above clearly.
2. Do not tear pages from this book; all your writing — even rough work — must be handed in. You may do rough work for this paper anywhere in the booklet.
3. Calculators are permitted, as long as they dont have any textual memory or graphing functions.
4. This examination booklet consists of this cover, Pages 1 through 8 containing questions; and Pages 9, 10, and 11, which are blank.
5. There are two kinds of problems on this examination, each clearly marked as to its type.
   - Most of the questions on this paper require that you SHOW ALL YOUR WORK! Their solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
   - Some of the questions on this paper require only BRIEF SOLUTIONS ; for these you are expected to write the correct answer in the box provided; you are not asked to show your work, and you should not expect partial marks for solutions that are not correct.

   You are expected to simplify your answers wherever possible.

   You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)
6. A TOTAL OF 100 MARKS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a) /5	1(b) /5	1(c) /5	1(d) /5	2(a) /5	2(b) /5	2(c) /5	3 /10	
	4(a) /5	4(b) /5	5(a) /5	5(b) /5	6(a) /6	6(b) /6	7(a) /6	7(b) /6
8(a) /5	8(b) /6							TOTAL /100

1. SHOW ALL YOUR WORK!

   (a) [5 MARKS] Evaluate $\displaystyle\int_1^2 \frac{2x+1}{2x^2+x+1}\,dx$.

   (b) [5 MARKS] Evaluate $\displaystyle\int x^2 \sin x\,dx$.

   (c) [5 MARKS] Evaluate $\displaystyle\int \frac{\cos x}{\sqrt{\sin x}}\,dx$.

   (d) [5 MARKS] Evaluate $\displaystyle\frac{d}{dx}\int_{x+1}^{x+3} e^{\sqrt{t}}\,dt$.

SHOW ALL YOUR WORK!

2. For each of the following series you are expected to apply one or more tests for convergence or divergence and determine whether the series is convergent. In each case you must answer 3 questions:

   - Name the test(s) that you are using.
   - Explain why the test(s) you have chosen is/are applicable to the given series.
   - Use the test(s) to conclude whether or not the series is convergent.

   (a) [5 MARKS] $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n^2 \ln n}$

   (b) [5 MARKS] $\displaystyle\sum_{n=2}^{\infty} 2^{1/(n^2-n)}$

   (c) [5 MARKS] $\displaystyle\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$

3. SHOW ALL YOUR WORK!

[10 MARKS] The region bounded by the x-axis and the curve

$$y = \sin(4x^2)$$

for $0 \leq x \leq \frac{1}{2}\sqrt{\pi}$ is rotated about the y-axis. Find the volume of the resulting solid.

4. BRIEF SOLUTIONS In part (b) of this question you are not requested to evaluate the resulting integral, but just to express your answer as a definite integral. It is not enough to quote a general formula: your integral must have integrand and limits specific to the given problem.

   (a) [5 MARKS] Draw the graph of the curve given in polar coordinates by $r = \frac{\pi}{\theta}$, for $\pi/2 \leq \theta \leq 6\pi$.

   YOUR DRAWING HERE:

   (b) [5 MARKS] Find the length of the above curve

   DEFINITE INTEGRAL ONLY (DO NOT EVALUATE)

5. SHOW ALL YOUR WORK!

   Check the convergence of the following integrals. Whenever the integral converges, evaluate it.

   - [5 MARKS] $\displaystyle\int_e^\infty \frac{dx}{x(\ln x)^2}$
   - [5 MARKS] $\displaystyle\int_0^{\pi/2} \frac{dx}{\cos^2 x}$

6. SHOW ALL YOUR WORK!

(a) [6 MARKS] Evaluate: $\lim_{n\to\infty} \dfrac{(n!)^2}{(2n)!}$

(b) [6 MARKS] Evaluate: $\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \sin(\pi i/n)$. Hint: use Riemann sums.

7. SHOW ALL YOUR WORK!

Consider the curve C defined by

$$\begin{aligned} x &= e^t - t \\ y &= e^t + t\,. \end{aligned}$$

(a) [6 MARKS] Find the tangent lines to C at $(1,1)$ and at $(e-1, e+1)$.

(b) [6 MARKS] Find the area of the region between the curve C and the x-axis for $1 \leq t \leq 3$.

8. SHOW ALL YOUR WORK!

(a) [5 MARKS] Find the sum

$$\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+7)}$$

Hint: Find a partial fractions decomposition, then expand and cancel out the expression for the nth partial sum.

(b) [6 MARKS] Given the series

$$\sum_{n=1}^{\infty} \frac{1}{(n+\frac{1}{2})^{1.1}} = S,$$

use the integral test to find an upper bound on the error term $S - S_n$.

CONTINUATION PAGE FOR PROBLEM NUMBER

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