

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-141A

CALCULUS II

Examiner: Dr B. ANCHOUCHE
Associate Examiner: Professor W. G. BROWN

Date: December, 13, 2000.
Time: 9:00 A.M. - 12:00 Noon

INSTRUCTIONS

Calculators are not permitted.

- This examination booklet consists of this cover, pages 2 through 10 containing questions, and pages 11 and 12, which are blank. For all problems you are expected to show all your work. All solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last two pages, or the back cover of the booklet, but you **must** indicate any continuation clearly on the page where the question is printed! You may do rough work anywhere in the booklet.

(Please inform the invigilator if you find that your booklet is defective.)

PLEASE DO NOT WRITE INSIDE THIS BOX

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|-----|------|------|-----|-----|-------|------|--|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| /10 | /10 | /5 | /10 | /10 | /10 | /15 | |
| 8 | 9 a) | 9 b) | | | | | |
| /10 | /5 | /5 | | | | | |
| | | | | | TOTAL | | |
| | | | | | /90 | /100 | |

Problem 1: (10 Marks) Showing all your work, evaluate the following integral:

$$\int x^5 \sqrt{4 - x^2} dx.$$

Problem 2: (10 Marks) Showing all your work, evaluate the following integral:

$$\int \frac{3e^{2x} + 2e^x}{9e^{2x} + 6e^x + 5} dx.$$

Hint: Use the substitution $u = e^x$.

Problem 3: (5 Marks) Find the values of p for which the following integral converges, and evaluate it for those values of p

$$\int_0^1 x^p (\ln x) dx.$$

Hint: Use integration by parts and the facts (without proof) that $\lim_{\alpha \rightarrow 0^+} \alpha^p (\ln \alpha) = 0$ if $p > 0$ and $\lim_{\alpha \rightarrow 0^+} \alpha^p (\ln \alpha) = -\infty$ if $p \leq 0$.

Problem 4: (10 Marks) Find the area of the surface obtained by rotating the curve

$$y = \frac{1}{2}(e^{-x} + e^x); \quad 0 \leq x \leq 1,$$

about the x-axis.

Problem 5: (10 Marks) Find the length of the curve

$$x = a(\cos t + t \sin t), \quad y = a(\sin t - t \cos t),$$

$0 \leq t \leq \pi$, where a is a positive real number.

Problem 6: (10 Marks) Use the method of cylindrical shells to find the volume of the solid obtained by rotating around the y -axis, the region bounded by the curves

$$y = \sin(x^2); \quad y = 0; \quad \text{for } 0 \leq x \leq \sqrt{\pi}.$$

Problem 7:(15 Marks) Find the vertices, foci, and asymptotes of the following *hyperbola*, and sketch its graph

$$5x^2 - 4y^2 - 40x + 24y = -24.$$

Problem 8: (10 Marks) Determine the values of p for which the following series converges:

$$\sum_{n=2}^{+\infty} \frac{1}{n (\ln n)^p}.$$

Problem 9: (10 Marks) Determine for each of the following series whether it is absolutely convergent, conditionally convergent, or divergent.

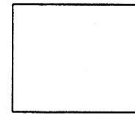
a)

$$\sum_{n=1}^{\infty} \frac{8n(-3)^n}{4^{n-1}}.$$

b)

$$\sum_{n=1}^{\infty} \frac{5n^n}{3^{3n+1}}.$$

CONTINUATION PAGE FOR PROBLEM NUMBER



You *must* refer to this continuation page on the page where the problem is printed!