VERSION 1

Student Name:

Student Number:

Faculty of Science
FINAL EXAMINATION

Mathematics 140
Calculus I
Monday April 16, 2007
2 pm – 5 pm

Examiner: Frederic Pollack

Associate Examiner: Professor W. G. Brown

This is a closed book examination.

On all problems, work must be shown to receive credit.

Answer directly on the exam. You are allowed to use the back page to complete your solutions.

Language dictionaries are allowed.

Calculators are not allowed.

There are 13 questions and 14 pages to this exam and the total number of marks for the exam is 100.

Final Examination Mathematics 140 VERSION 1

1) (15 points). Evaluate the following limits, if they exist. If the limit is $+\infty$ or $-\infty$, write $+\infty$ or $-\infty$ respectively. In all other cases, write "NO FINITE OR INFINITE LIMIT". Show all work.

a) (4 points). $\lim_{x \to 0^+} \frac{\sin 2x}{\sqrt{x}}$

b) (4 points). $\lim_{x \to -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$

c) (4 points). $\lim_{x\to 0}(\cos x)^{1/x^2}$

d) (3 points). $\lim_{x\to 2}\frac{\sqrt{x^2-4}}{2}$

2) (8 points). Differentiate. Simplify, when possible. Show all work.

a) (4 points). $y = \tan^3\left(e^{\sqrt{\ln x}}\right)$

b) (4 points). $f(x) = x^{2/x}$

3) (7 points). Find the equation of the tangent line to the curve at the given point. Show all work.

$$y = \frac{x}{\sqrt{x+3}} \qquad (1, 1/2)$$

4) (7 points). A snowball melts so that its surface area decreases at a rate of $1\text{cm}^2/\text{min}$, find the rate at which the radius r decreases when the radius r is 5 cm? The surface area of a sphere (snowball) is $A = 4\pi r^2$. Show all work.

5) (7 points). Consider the family of cubic polynomials $y = Ax^3 + 6x^2 - Bx$, where A and B are unknown constants. Either 1) determine the value of A and B so that the graph of y has a maximum value at $x = -1$ and an inflection point at $x = 1$; or 2) prove that no member of the family has the desired properties.

6) (7 points). If 1200 cm^2 of material is available to make a box with a square base and open top, find the dimensions that would maximize the volume of the box. Show all work.

7) (14 points). Let $f(x) = \ln(x^2 + 4)$.

Show all your work and use calculus to answer the following questions.

a) (4 points). Find the intervals where the function is increasing and the intervals where the function is decreasing. Show all work.
b) (4 points). Find the local minimum point(s) and local maximum point(s) of the function, if any. Show all work.
c) (4 points). Find the inflection point(s) of f, if any. Show all work.
d) (2 points). Sketch the graph of the function.

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8) (7 points). Let $g(x) = |3 - x|$. Either prove that g is not differentiable at x = 3, or use the limit definition of derivative to determine the value of $g'(3)$.

9) (5 points). Find the antiderivative F(x) of $f(x) = \dfrac{1}{\sqrt{1-x^2}}$ such that $F(0) = 5$.

Show all work.

10) (6 points). Prove the identity $\frac{1+\tanh x}{1-\tanh x} = e^{2x}$. Show all work.

11) (5 points). Find the linearization L(x) of the function $f(x) = \sqrt[3]{1+2x}$ at a = 0.
Show all work.

12) (6 points). Determine the real constants a, b such that $f(x) = \begin{cases} x^2 + bx + 1 & x < 5 \\ 8 & x = 5 \\ ax + 3 & x > 5 \end{cases}$

is continuous on $(-\infty, \infty)$. Show all work.

13) (6 points). It is known that x and y are related by the equation $x^2y^2(1+xy)+4=0$. Using any valid method, determine $\frac{dy}{dx}$ in terms of x and y. After that, determine $\frac{dy}{dx}$ at the point $(1, -2)$.