

Student Name:

Student ID#

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 140

Calculus I

Examiner: Professor J. Loveys
Associate Examiner: Professor W. Brown

Date: Wednesday April 12, 2006.
Time: 2:00 PM - 5:00 PM.

INSTRUCTIONS

1. Please clearly print your name and student ID# where indicated at the top of this page.
2. Do not remove any pages from this exam.
3. Each of the five parts of problem 1 is worth 4 marks. **There will be no part marks for any of these** (i.e., you will receive 0 or 4 for 1(a), etc.) You need not show your work for problem 1.
4. For problems 2 through 8, **you must show your work**. Put your final answer where it says "ANSWER:" for each of these problems.
5. Calculators are not permitted.
6. This is a closed book exam. Notes and books are not permitted.

7. *Regular and translation dictionaries are allowed.*

This exam comprises the cover page, 9 pages of questions and 3 blank pages for rough or extra work. You should indicate clearly on the page where the problem is stated if you have work you want considered on any of these pages. Good Luck, and have fun!

1. Evaluate each of the following limits, if they exist. Give the number, if the limit is a number; if the limit is ∞ or $-\infty$, clearly write ∞ or $-\infty$ (whichever is correct). If none of the above is the case, clearly PRINT "NO FINITE OR INFINITE LIMIT EXISTS." (4 marks each; no part marks.)

(a)

$$\lim_{v \rightarrow \infty} (v - \sqrt{v^4 + 4v}).$$

ANSWER:

(b)

$$\lim_{x \rightarrow 0} \frac{x^2 + 1}{1 - \cos 2x}.$$

ANSWER:

(c)

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{|x - 3|}.$$

ANSWER:

(d)

$$\lim_{u \rightarrow \ln 2} \frac{e^{2u} + e^u - 6}{e^{2u} + 6e^u - 16}.$$

ANSWER:

(e)

$$\lim_{x \rightarrow -\infty} \frac{\cosh 2x + \sinh 2x}{(\cosh x + \sinh x)^2}.$$

ANSWER:

2. (10 marks) Find all solutions θ with $0 \leq \theta \leq 2\pi$ of the following equation.

$$2 \cos^2 \theta + \sin \theta = 1.$$

YOU ARE EXPECTED TO SHOW ALL YOUR WORK.

ANSWER:

WORK:

3. (10 marks) Find $f'(x)$ if

$$f(x) = \arcsin(e^{\tan x}).$$

[Don't bother simplifying your answer; just get it right.] YOU ARE EXPECTED TO SHOW ALL YOUR WORK.

ANSWER:

WORK:

4. (10 marks) Suppose that $y = f(x)$ is defined implicitly by

$$y^3 - 3xy^2 + 24x = 8.$$

Find $f(0)$, $y' = f'(x)$ (in terms of x and y) and $f'(0)$. YOU ARE EXPECTED TO SHOW ALL YOUR WORK.

ANSWERS:

WORK:

5. (10 marks) The surface area of your beautiful spherical balloon is increasing at the constant rate of $5\text{cm}^2/\text{sec}$. How fast is the volume increasing when the radius is 10cm ?

[Recall that the surface area of a sphere is given by $A = 4\pi r^2$ and the volume by $V = \frac{4}{3}\pi r^3$ if r is the radius. You may want to compute $\frac{dr}{dt}$ along the way.] YOU ARE EXPECTED TO SHOW ALL YOUR WORK.

ANSWER:

WORK:

6. (10 marks) You want your tin cans to be cylinders. (So the bottom and top are circular disks of the same radius r , and the side has area $2\pi r h$ if r is the radius of the bottom and h the height of the can.) You want them to hold exactly $54\pi \text{cm}^3$ of soup. You also want to use as little tin as possible on each can. What should r and h be?

[Reminder: the volume is given by $V = \pi r^2 h$.] YOU ARE EXPECTED TO SHOW ALL YOUR WORK.

ANSWER:

WORK:

7. (20 marks total.) Sketch the graph of the following function f .

$$f(x) = 3 - \frac{15}{x^2 - 4}.$$

I'm not expecting great art, but clearly answer the following. Put your graph and work on the following page.

(a) (1 mark) What is the domain of f ?

ANSWER:

(b) (2 marks) For which value(s) of x is $f(x) = 0$?

ANSWER:

(c) (3 marks) What are the asymptotes (vertical, horizontal, and/or slant) of the graph? (If there aren't any, print "NONE".)

ANSWER:

(d) (3 marks) Where are the local maximum(s) and minimum(s), if any?

[Give me both a and $f(a)$ if the local maximum or minimum occurs at $x = a$. Tell me which it is — maximum or minimum. If there aren't any, print "NONE".]

ANSWER:

(e) (3 marks) On which intervals is f increasing, and on which decreasing?

ANSWER:

(f) (3 marks) Where is the graph concave up, and where concave down?

ANSWER:

(g) (2 marks) Where are the inflection points (if any)? (If there aren't any, print "NONE".)

ANSWER:

YOU ARE EXPECTED TO SHOW ALL YOUR WORK. GRAPH (3 marks) and WORK for problem 7:

8. (10 marks) Find the general antiderivative $F(x)$ of the function f defined by

$$f(x) = 4x^3 + \frac{3}{1+x^2}.$$

YOU ARE EXPECTED TO SHOW ALL YOUR WORK.

ANSWER:

WORK:

This is an extra page for rough or extra work. Please indicate which problem(s) you are using this page for ON THE PAGE WHERE THE PROBLEM(S) IS/ARE STATED.

This is an extra page for rough or extra work. Please indicate which problem(s) you are using this page for ON THE PAGE WHERE THE PROBLEM(S) IS/ARE STATED.

This is an extra page for rough or extra work. Please indicate which problem(s) you are using this page for ON THE PAGE WHERE THE PROBLEM(S) IS/ARE STATED.