Faculty of Science

FINAL EXAMINATION

MATH 140

Calculus I

Examiner: Dr. D. Serbin
Associate Examiner : Prof. W. Brown

Date: Monday, April 18, 2005
Time: 9 A.M. - 12 Noon

INSTRUCTIONS

There are 9 questions, altogether worth 110 marks.
Answer all questions in examination booklets.
Show all your work. Simplify your final answers.
This is a closed book examination.
Calculators are not permitted.
You may keep the exam paper when finished.

This exam comprises the cover and 2 pages of questions.

1. Evaluate the following limits. **You may use l'Hospital's rule only in (c).**

(a) [5 MARKS]

$$\lim_{x\to -3} \frac{x+3}{x^2+2x-3}$$

(b) [5 MARKS]

$$\lim_{x\to\infty} (\sqrt{x+2}-\sqrt{x}),$$

(c) [7 MARKS]

$$\lim_{x\to 0} \left(\frac{4-2x^2-4\cos x}{x^4} \right).$$

2. Compute the derivative of each of the following functions.

(a) [5 MARKS]

$$f(x) = \cosh(\sqrt{x^2-1}),$$

(b) [5 MARKS]

$$g(x) = \frac{\sin x}{e^x}.$$

(c) [5 MARKS]

$$h(x) = x^{x^2+1}.$$

3. A function $f(x)$ is defined as follows

$$f(x) = \begin{cases} x^2 - 2x - a & \text{if } x < 1, \\ -x^2 + bx + 2 & \text{if } x \geq 1, \end{cases}$$

where a and b are constants to be evaluated.

(a) [5 MARKS] Showing all your work determine all values of a and b which make f continuous at $x = 1$.

(b) [5 MARKS] Showing all your work determine all values of a and b which make f differentiable at $x = 1$.

4. [10 MARKS] A particle is moving along the parabola $y = (x-1)^2$ in such a way that its x-coordinate is increasing at the constant rate of 4 units per second. Determine how fast the distance from the particle to the origin is changing at the instant the particle is at the point $(2, 1)$.

(continued on the next page)

5. (a) [5 MARKS] Find the function f that satisfies all of the following conditions:

$$f''(x) = 2e^x - \cos x, \quad f(0) = 1, \quad f'(0) = 1.$$

(b) [5 MARKS] Use the Mean Value Theorem to show that

$$\ln\ x < x - 1,$$

when $x > 1$.

6. The equation

$$(x + y)^3 = 27(x - y)$$

defines y implicitly as a function of x near the point $(2, 1)$.

(a) [7 MARKS] Determine the values of y' and y'' at $(2, 1)$.

(b) [3 MARKS] Use the tangent line approximation to estimate y when $x = 1.98$.

7. Let

$$f(x) = \frac{(x-1)^2}{e^x}.$$

(a) [3 MARKS] Specify the domain of f.

(b) [5 MARKS] Determine all horizontal and vertical asymptotes.

(c) [5 MARKS] Determine the interval(s) where f increases, the interval(s) where f decreases, and local extrema.

(d) [5 MARKS] Determine the concavity of the graph of f, and find all points of inflection. **(Hint: You might need the fact $\sqrt{2} \approx 1.4$).**

(e) [4 MARKS] Sketch the graph of f, using the information determined above.

8. [8 MARKS] Showing all your work, determine the maximum area of a rectangle with a base that lies on the x-axis, and two upper vertices that lie on the graph of the equation $y = 2 - x^2$.

9. [8 MARKS] Showing all your work, determine the absolute maximum and minimum values of

$$f(x) = \frac{\sin x}{2 - \cos x}$$

on the closed interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. **(Hint: $\sqrt{3} \approx 1.7$)**