McGill University — Faculty of Science

FINAL EXAMINATION

MATHEMATICS 140 2010 09   CALCULUS 1

EXAMINER: Professor S. W. Drury   DATE: Wednesday, December 8, 2010
ASSOCIATE EXAMINER: Dr. A. Hundemer   TIME: 9:00 am. – noon

FAMILY NAME:

GIVEN NAMES:

MR, MISS, MS, MRS, &c.:   STUDENT NUMBER:

INSTRUCTIONS

1. Fill in the above clearly.

2. Do not tear pages from this book; all your writing — even rough work — must be handed in. You may do rough work anywhere in the booklet except in the box below and in the answer boxes.

3. This is a closed book examination. Calculators are not permitted, but regular and translation dictionaries are permitted.

4. This examination consists of two parts, Part A and Part B. READ CAREFULLY THE MORE DETAILLED INSTRUCTIONS AT THE START OF EACH PART.

5. Part A has 20 multiple choice questions each of which is worth 2 points for a total of 40 points. It is recommended that in the first instance, you do not spend more than 4 minutes per question or 75 minutes in total on Part A.

6. Part B has five questions worth a total of 46 points which should be answered on this question paper. The questions in part B are of two types:

   - In BRIEF SOLUTIONS questions, each answer will be marked right or wrong. Within a question, each answer has equal weight.

   - In SHOW ALL YOUR WORK questions a correct answer alone will not be sufficient unless substantiated by your work. Partial marks may be awarded for a partly correct answer.

   In Part B, you are expected to simplify all answers wherever possible.

7. This examination booklet consists of this cover, Pages 1–3 containing questions in Part A, Pages 4–9 containing questions in Part B and Pages 10–13 which are blank continuation pages.

8. A TOTAL OF 86 POINTS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

code	21	22	BS total
	/8	/8	/16
23	24	25	SA total
/10	/10	/10	/30

Part A: MULTIPLE CHOICE QUESTIONS

Each of the following 20 questions is worth 2 points. The maximum number of points you may earn on these multiple choice questions is 40 points. There is only one correct answer expected for each question. The questions are to be answered on the answer card provided. Be sure to enter on the answer card:

- ○ Your student number. ○ The version of the exam that you are taking, i.e. version 1.
- ○ Your name. ○ The check code, i.e. the first two letters of your family name.

Fill in the disks below your student number, check code and version number.
It is recommended that in the first instance, you do not spend more than 4 minutes per question or 75 minutes in total on Part A.
Please note:

> The Examination Security Monitor Program detects pairs of students with un-usually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, either to initiate or corrob-orate an investigation or a charge of cheating under Section 16 of the Code of Student Conduct and Disciplinary Procedures.

1. (2 points)   Find $\displaystyle\lim_{x\to-\infty}\frac{\ln(6e^{12x}+6x^2)}{\ln(2e^{4x}+3x^2)}$.

    (a) 1,   (b) ln(2),   (c) 3,   (d) ln(3),   (e) does not exist.

2. (2 points)   Find $\displaystyle\lim_{x\to0}\frac{1-\cos(x)}{x}$.

    (a) 1,   (b) 0,   (c) 2,   (d) 1/2,   (e) does not exist.

3. (2 points)   Find $\displaystyle\lim_{x\to+\infty}\frac{\sqrt{25+2x}-5}{x}$.

    (a) 0,   (b) ∞,   (c) 2/5,   (d) 1/5,   (e) does not exist.

4. (2 points)   The function defined by

$$f(x)=\begin{cases} x+3 & \text{if } x<-2, \\ -3-x & \text{if } -2\le x\le 5. \\ 2x-18 & \text{if } 5<x. \end{cases}$$

fails to be continuous

    (a) only at $x=-2$,   (b) only at $x=5$,   (c) only at $x=-2$ and $x=5$,
                    (d) nowhere,   (e) on some other set.

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5. (2 points)   Let $f(x) = 4x^2 + 2x + 3$. Suppose that the Mean Value Theorem applied to f on the interval $[3, X]$, can be used to establish the existence of a number x satisfying $3 < x < X$ and such that $f'(x)$ equals 38. Then X equals

$$\text{(a) 7,} \qquad \text{(b) 10,} \qquad \text{(c) 13,} \qquad \text{(d) 9,} \qquad \text{(e) 6.}$$

6. (2 points)   Let $f(x) = 5x^3 + x^2 + x + 3$. The Intermediate Value Theorem applied to f on the interval $[-2, 2]$, can be used to establish the existence of a number x_0 satisfying $-2 < x_0 < 2$ and such that $f(x_0)$ equals

$$\text{(a) } -39, \qquad \text{(b) 57,} \qquad \text{(c) 51,} \qquad \text{(d) 54,} \qquad \text{(e) } -9.$$

7. (2 points)   The function defined by

$$f(x) = \begin{cases} \dfrac{\sin(6x)}{2x} & \text{if } x < 0, \\[2mm] a\cos(4x) & \text{if } x \geq 0. \end{cases}$$

is continuous if and only if a is

$$\text{(a) 3,} \qquad \text{(b) } -1, \qquad \text{(c) 4,} \qquad \text{(d) 6,} \qquad \text{(e) 0.}$$

8. (2 points)   Let $f(x) = x^4 \ln(x^2 + 9)$. Then $f'(2)$ is

(a) $32\ln(13)$,   (b) $32\ln(13) + 16$,   (c) $32\ln(13) + 16/13$,   (d) $32\ln(13) + 4/13$,
(e) $32\ln(13) + 64/13$.

9. (2 points)   The horizontal asymptote(s) of $f(x) = \dfrac{12x^3 + 3x}{\sqrt{9x^6 + 2}}$ is (are) best described by

(a) $y = 4$,   (b) $y = 12$,   (c) $y = 4/3$,   (d) two asymptotes,   (e) no asymptote.

10. (2 points)   How many vertical asymptotes does the function $f(x) = \ln\left(\left|\dfrac{x+1}{x^2 - 7x + 12}\right|\right)$ have

(a) none,   (b) one,   (c) two,   (d) three,   (e) four.

11. (2 points)   The range of the function $x \mapsto \tanh(x)$ is

(a) $[1, \infty)$,   (b) $[0, 1]$,   (c) $(-1, 1)$,   (d) $(-\infty, \infty)$,   (e) $[0, \infty)$.

12. (2 points)   The tangent to the curve $y = 2x^3 + 4x$ at $(-2, -24)$ passes through the point

(a) $(1, 63)$,   (b) $(2, 88)$,   (c) $(0, 35)$,   (d) $(-1, 8)$,   (e) $(4, 146)$.

13. (2 points)   Let $y = f(x)$ be defined by the equation $5\sin(x) + 2\cos(y) = \sqrt{3}$ near $x = 0$, $y = \pi/6$. The value of $f'(0)$ is

$$\text{(a) } 16, \quad \text{(b) } 5, \quad \text{(c) } 20, \quad \text{(d) } 10, \quad \text{(e) } -2.$$

14. (2 points)   The absolute maximum value taken by the function $f(x) = 5\sin(x) - 2\cos(x)$ on $[0, \frac{\pi}{2}]$ is

$$\text{(a) } 5, \quad \text{(b) } \sqrt{29}, \quad \text{(c) } 4, \quad \text{(d) } 13, \quad \text{(e) } 3.$$

15. (2 points)   Let f denote the function $f(x) = (1 + x^4)e^{-3x}$ defined on $[0, \infty)$. Which answer best describes the location where f takes its absolute minimimum value?

$$\text{(a) } x = 2, \quad \text{(b) } x = 14, \quad \text{(c) } x = 0, \quad \text{(d) } x = 1/3, \quad \text{(e) minimum not attained.}$$

16. (2 points)   Let $f(x) = x^3 \ln(x)$. Then $f''(1)$ is

$$\text{(a) } 11, \quad \text{(b) } -2, \quad \text{(c) } 12, \quad \text{(d) } 3, \quad \text{(e) } 5.$$

17. (2 points)   Let $f(x) = x^3 - 3x^2 - 189x + 2$. The function f has a point of inflection at x equals

$$\text{(a) } 9, \quad \text{(b) } 1, \quad \text{(c) } -7, \quad \text{(d) } -1, \quad \text{(e) } -6.$$

18. (2 points)   Let $f(x) = \arctan(x^6)$. Then $f'(1)$ is

$$\text{(a) } 3, \quad \text{(b) } 17, \quad \text{(c) } 13, \quad \text{(d) } 14, \quad \text{(e) } 6.$$

19. (2 points)   Let $f(x) = \ln\left(\dfrac{x^6}{x^8 + 1}\right)$. Then $f'(1)$ is

$$\text{(a) } 6, \quad \text{(b) } 2, \quad \text{(c) } -2, \quad \text{(d) } 10, \quad \text{(e) } 5.$$

20. (2 points)   Let $f(x) = (3x + 9)e^{-x}$. The largest interval containing $x = 12$ on which f is concave up is

$$\text{(a) } [11, \infty), \quad \text{(b) } [-2, 25], \quad \text{(c) } [-2, \infty), \quad \text{(d) } [-1, \infty), \quad \text{(e) } [-1, 25].$$

Part B: ANSWER DIRECTLY ON THE QUESTION PAPER

In Part B there are 5 questions each worth a total of 46 points. In Part B, you are expected to <u>simplify all answers</u> wherever possible.

The questions in part B are of two types:

- In BRIEF SOLUTIONS questions worth 8 points each, each answer will be marked right or wrong. Within a question, each answer has equal weight. For these questions start your rough work below the answer box and use the facing page if necessary.

- In SHOW ALL YOUR WORK questions worth 10 points each, a correct answer alone will not be sufficient unless substantiated by your work. Partial marks may be awarded for a partly correct answer. Begin your solution on the page where the question is printed. You may continue a solution *on the facing page*, or on the continuation pages, or the back cover of the booklet, but you <u>must</u> indicate any continuation clearly on the page where the question is printed! Write your final answer in the answer box provided.

21. (8 points) BRIEF SOLUTIONS At noon, ship A is 50 km. west of ship B. Ship A is sailing east at 20 km./hr. and Ship B is sailing north at 10 km./hr. What is the distance in km. between the two ships t hours after noon?

ANSWER ONLY

At what rate is the distance between the ships increasing at 3 pm. in km./hr.?

ANSWER ONLY

22. (8 points) BRIEF SOLUTIONS Let $f(x) = x^2 - 24x + 18\ln(x)$ defined for $x > 0$.

ANSWER ONLY

$f'(x) =$

ANSWER ONLY

$f''(x) =$

The interval on which f is decreasing is

ANSWER ONLY

The graph of $y = f(x)$ has an inflection point where

ANSWER ONLY

$x =$

If there appears to be more than one answer, recall that $f(x)$ is defined only for $x > 0$.

23. (10 points) SHOW ALL YOUR WORK!

The base of a box with an open top must have a width of 4 cm., but the length x cm. of the base is unrestricted. The material for the base costs \$6 per square cm. and that for the sides costs \$3 per square cm. If the volume of the box is to be 84 cubic cm., what is the cost of the box in dollars as a function of x?

ANSWER ONLY

What is the most economical length for the base in cm.?

ANSWER ONLY

24. (10 points) SHOW ALL YOUR WORK!
    Let $f(x) = |x^2 - 5x - 6| + 2x^2 + 17x$.
    Find the absolute maximum and absolute minimum values of $f(x)$ for $-6 \le x \le 3$ and the locations where they are attained.

ANSWER ONLY			
Absolute maximum value =		taken at $x =$	
Absolute minimum value =		taken at $x =$	

25. (10 points) SHOW ALL YOUR WORK!
    Using L'Hospital's Rule or otherwise determine

$$\lim_{x \to 1} \left(\frac{1}{\ln(x)} - \frac{x^6}{x-1} \right).$$

ANSWER ONLY