VERSION **1**

McGILL UNIVERSITY — FACULTY OF SCIENCE
FINAL EXAMINATION

MATHEMATICS 140 2009 09 CALCULUS 1

EXAMINER: Professor W. G. Brown
ASSOCIATE EXAMINER: Dr. A. Hundemer

DATE: Wednesday, December 16th, 2009
TIME: 14:00 – 17:00 hours

FAMILY NAME:
GIVEN NAMES:

STUDENT NUMBER

Instructions

1. Do not tear pages from this book; all your writing — even rough work — must be handed in. You may do rough work for this paper anywhere in the booklet.
2. This is a closed book examination. Calculators are not permitted, but regular and translation dictionaries are permitted.
3. This examination booklet consists of this cover, Pages 1–4 containing *multiple-choice* questions worth at most 50 MARKS, Pages 5–8 containing *full solution* questions worth 30 MARKS, and Pages 9–13 which are blank continuation pages. A TOTAL OF 80 MARKS ARE AVAILABLE ON THIS EXAMINATION.
   - Your answers to the *multiple choice* questions must be entered on the Scantron form which will be provided. There is only 1 correct answer expected for each problem. Be sure to enter (1) Your student number (including the *check code* = the last two letters of your family name), (2) Your examination Version, (3) Your name. Please note that the Examination Security Monitor Program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, either to initiate or corroborate an investigation or a charge of cheating under Section 16 of the Code of Student Conduct and Disciplinary Procedures.
   - *Full solution* questions in the second part of this paper require that you SHOW ALL YOUR WORK! Begin your solution on the page where the question is printed; a correct answer alone will not be sufficient unless substantiated by your work. You may continue a solution *on the facing page*, or on the continuation pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed! *To be awarded partial marks on a part of a* full solution *question a student's answer for that part must be deemed to be more than 50% correct.* You are expected to simplify all answers wherever possible.

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1(a) /4	1(b) /4	1(c) /7	2 /6	3 /9							
										TOTAL NMC /30	

PART I: MULTIPLE CHOICE QUESTIONS

Each of the following 30 questions is worth 2 MARKS. The maximum number of marks you may earn on these multiple choice questions is 50 MARKS; you may attempt as many of these problems as you wish. Show your answers only on the Scantron form.

1. The function defined by

$$f(x) = \begin{cases} x+4 & \text{if } x < -4, \\ -4-x & \text{if } -4 \le x \le 5. \\ 2x-21 & \text{if } 5 < x. \end{cases}$$

fails to be continuous

(a) only at $x = -4$, (b) only at $x = 5$, (c) only at $x = -4$ and $x = 5$, (d) nowhere, (e) on some other set.

2. Let $f(x) = x^5 \ln(x)$. Then $f''(1)$ is

(a) 7, (b) 5, (c) 9, (d) 3, (e) 2.

3. The vertical asymptote(s) of $f(x) = \dfrac{\sqrt{2x^4+4}}{x^2+2x-3}$ is (are) best described by

(a) $x = -9$, (b) $x = 0$, (c) $x = -3$, (d) two asymptotes, (e) no asymptote.

4. Let $y = f(x)$ be defined by the equation $14x + yx^2 + y^3 = -16$ near $x = -1$, $y = -1$. The value of $f'(-1)$ is

(a) -5, (b) -7, (c) -4, (d) -11, (e) -9.

5. Let

$$f(x) = \begin{cases} \dfrac{(\sin(x))^5}{x^4} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Then $f'(0)$ is

(a) -1, (b) 0, (c) 4, (d) 1, (e) does not exist.

6. Find $\displaystyle\lim_{x\to 0} \frac{1-\cos(x)}{x^3}$.

(a) 1/2, (b) 1, (c) 2, (d) 0, (e) does not exist.

7. The minimum value taken by the function $f(x) = 3\sin(x) - 2\cos(x)$ on $[0, \frac{\pi}{2}]$ is

(a) -2, (b) -4, (c) 5, (d) -12, (e) -10.

8. Let $f(x) = x^4 \ln(x^2 + 7)$. Then $f'(2)$ is

(a) $32\ln(11) + 4/11$, (b) $32\ln(11) + 16/11$, (c) $32\ln(11)$, (d) $32\ln(11) + 64/11$, (e) $32\ln(11) + 16$.

9. The normal to the curve $y = \dfrac{4x}{1+x}$ at $(1, 2)$ passes through the point

(a) $(-2, 6)$, (b) $(-7, 8)$, (c) $(0, 4)$, (d) $(4, -1)$, (e) $(-6, 10)$.

10. Let $f(x) = x\sin(x) + \cos(x)$. The number of values of x in the range $[0, 30\pi]$ at which f has a local maximum is

(a) 16, (b) 15, (c) 0, (d) 30, (e) 29.

11. Find $\displaystyle\lim_{x\to -3} \frac{x^2 + 9x + 18}{x^2 + 7x + 12}$.

(a) 0, (b) -2, (c) 2, (d) 3, (e) does not exist.

12. Let $y = f(x)$ be defined by the equation $y + 12x^2 + xe^{4y} = 13$ near $x = 1$, $y = 0$. The value of $f'(1)$ is

(a) -9, (b) -7, (c) -5, (d) -10, (e) -13.

13. Let $f(x) = \arctan(x^6)$. Then $f'(1)$ is

(a) 6, (b) 3, (c) -4, (d) -3, (e) -11.

14. The function defined by

$$f(x) = \begin{cases} \dfrac{\sin(8x)}{2x} & \text{if } x < 0, \\ a\cos(5x) & \text{if } x \geq 0. \end{cases}$$

is continuous if and only if a is

(a) 5, (b) 8, (c) 4, (d) -1, (e) 0.

15. Let $f(x) = \arcsin(x)$. Then $f''\left(-\frac{4}{5}\right)$ is

(a) $-56/27$, (b) $-100/27$, (c) $-50/27$, (d) $95/27$, (e) $115/27$.

16. Let $f(x) = \frac{x}{x^2+5}$. Then $f'(3)$ is

(a) $-13/196$, (b) $-11/98$, (c) $-2/7$, (d) $5/196$, (e) $-1/49$.

17. Let $f(x) = x^3 - 9x^2 - 273x - 2$. The function f has a point of inflection at $x =$

(a) -1, (b) 3, (c) 4, (d) 10, (e) 13.

18. Find $\lim_{x \to 0} \frac{\sin(x)}{|x|}$.

(a) $1/2$, (b) 1, (c) -1, (d) 0, (e) does not exist.

19. Find $\lim_{x \to \infty} \frac{\sin(x^7)}{x^4}$.

(a) 0, (b) 4, (c) 3, (d) -5, (e) does not exist.

20. Let f denote the function $f(x) = (1+4x)e^{-9x}$ defined on $[0, \infty)$. Which answer best describes the location where f takes its global (=absolute) maximum value?

(a) $x = 5/36$, (b) $x = 0$, (c) $x = 4/9$, (d) $x = 1/9$, (e) maximum not attained.

21. Let $f(x) = \sin(x)$. The largest interval containing $x = -\frac{101}{7}\pi$ on which f is concave up is

(a) $[-14\pi, -13\pi]$, (b) $[0, \infty)$, (c) $[-16\pi, -14\pi]$, (d) $[-15\pi, -13\pi]$, (e) $[-15\pi, -14\pi]$.

22. The tangent to the curve $y = \frac{x^2-4}{2+x}$ at $(-1, -3)$ passes through the point

(a) $(-10, -16)$, (b) $(3, -3)$, (c) $(-5, -11)$, (d) $(-7, -7)$, (e) $(-2, -4)$.

23. Let $f(x) = \ln\left(\frac{5}{x} + 6\right)$. Then $f'(3)$ is

(a) $-\frac{1}{69}$, (b) $\frac{3}{23}$, (c) $\frac{5}{69}$, (d) $-\frac{6}{23}$, (e) $-\frac{5}{69}$.

24. Let $y = f(x)$ be defined by the equation $y^2 + 36x + 7x^3 \ln(y) = 37$ near $x = 1$, $y = 1$. The value of $f'(1)$ is

(a) -9, (b) -7, (c) -11, (d) -4, (e) -10.

25. Find $\lim_{x \to 0} \dfrac{4^x - 1}{3^x - 1}$.

(a) $\log_3 4$, (b) $\ln(4) - \ln(3)$, (c) 1, (d) $3\ln(4) - 4\ln(3)$, (e) $4/3$.

26. Let f denote the function $f(x) = \dfrac{x^2}{x + 14}$ defined on $[0, \infty)$. Which answer best describes the location where f takes its global (=absolute) minimum value?

(a) $x = 14$, (b) $x = 0$, (c) $x = 11$, (d) $x = 28$, (e) minimum not attained.

27. Find $\lim_{x \to \infty} \dfrac{\sqrt{9 + 8x} - 3}{x}$.

(a) 0, (b) ∞, (c) $4/3$, (d) $8/3$, (e) does not exist.

28. The graph of the function $f(x) = e^x \cdot (x^3 - 6x + 12)$ has all of its inflection points at

(a) $x = 0$, (b) $x = -6$, (c) $x = 0$ and $x = -6$, (d) $x = 6$, (e) none of the preceding.

29. For the function $f(x) = x \cdot e^{-\frac{x^2}{2}}$ the intervals of decrease are

(a) $(-1, 1)$, (b) $(-\infty, -1)$, (c) $(1, +\infty)$,

(d) $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, +\infty)$, (e) $(-\infty, -1)$ and $(1, +\infty)$.

30. The graph of the function $f(x) = x^{\frac{5}{3}} \cdot (x - 4)$ is concave downward on the interval

(a) $(-\infty, 1)$, (b) $(1, +\infty)$, (c) $(0, 1)$, (d) $(-\infty, 0)$, (e) $(0, +\infty)$.

PART II: FULL SOLUTION QUESTIONS

These questions are together worth 30 MARKS. Begin each solution on the page where the question is printed; a correct answer alone will not be sufficient unless substantiated by your work. You may continue a solution *on the facing page*, or on the continuation pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed! *To be awarded partial marks on a part of a question a student's answer for that part must be deemed to be more than 50% correct.* You are expected to simplify all answers wherever possible.

1. SHOW ALL YOUR WORK!

   (a) [4 MARKS] Let $g(x) = 1 + \sinh 2x$. Showing all your work, find a linearization of g at $a = 0$.

   (b) [4 MARKS] Showing your work, use your linearization to approximate $g(0.005)$.

   (c) [7 MARKS] Use either the Mean Value Theorem or Rolle's Theorem — no other method is acceptable — to explain why the graph of g crosses the line $y = 1$ exactly once.

2. SHOW ALL YOUR WORK!

[6 MARKS] Showing all your work, determine the function f such that $f''(x) = 2 - 12x$, $f(0) = 9$, $f(2) = 15$.

3. SHOW ALL YOUR WORK!

[9 MARKS] Showing all your work, find the point(s) on the curve $y = 2\sqrt{x}$ which is (are) closest to the point $(2, 8)$.

CONTINUATION PAGE FOR PROBLEM NUMBER

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