VERSION 1

McGILL UNIVERSITY
FACULTY OF SCIENCE
FINAL EXAMINATION

MATHEMATICS 140 2007 09 CALCULUS I

EXAMINER: Professor W. G. Brown
ASSOCIATE EXAMINER: Dr. D. Serbin

DATE: Thursday, December 06th, 2007
TIME: 09:00 – 12:00 hours

FAMILY NAME:
GIVEN NAMES:

STUDENT NUMBER:

INSTRUCTIONS

1. Fill in the above clearly.
2. DO NOT TEAR PAGES FROM THIS BOOK! All your writing — even rough work — must be handed in. You may do rough work anywhere in the booklet.
3. This is a CLOSED BOOK examination. CALCULATORS ARE NOT PERMITTED. Translation dictionaries are permitted; no other dictionaries are permitted.
4. OTHER CALCULUS EXAMINATIONS ARE BEING WRITTEN AT THIS TIME. THIS IS THE EXAMINATION IN MATH 140 ONLY!
5. The examination booklet consists of this cover, Pages 1 through 7 containing questions; and Pages 8, 9, and 10, which are blank. *Your neighbour's version may not be the same as yours.*
6. There are two kinds of problems on this examination, each clearly marked as to its type.
   - Some of the questions on this paper require that you SHOW ALL YOUR WORK! Their solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
   - Some of the questions on this paper require only BRIEF SOLUTIONS ; for these you are expected to write the correct answer in the box provided; you are not asked to show your work, and you should not expect partial marks for solutions that are not completely correct.

   You are expected to simplify your answers wherever possible.

   You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)
7. A TOTAL OF 70 MARKS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a) /2	1(b) /2	1(c) /2	1(d) /2	1(e) /2	2(a) /3	2(b) /3	2(c) /3	
3(a) /3	3(b) /3	3(c) /3	3(d) /3	4(a) /6	4(b) /4	5(a) /3	5(b) /3	5(c) /3
6 /10	7(a) /3	7(b) /3	7(c) /3	7(d) /1				TOTAL /70

1. BRIEF SOLUTIONS

[2 MARKS EACH] Give the numeric value of each of the following limits if it exists; if the limit is $+\infty$ or $-\infty$, write $+\infty$ or $-\infty$ respectively. In all other cases write "NO FINITE OR INFINITE LIMIT".

(a) $\lim_{x\to\infty} \dfrac{x+x^2}{1-2x^2} =$

ANSWER ONLY

(b) $\lim_{x\to 0} \dfrac{\sin\left(3x^2\right)}{(\sin 3x)^2} =$

ANSWER ONLY

(c) $\lim_{x\to 0^+} \arctan\left(-\dfrac{1}{x}\right) =$

ANSWER ONLY

(d) $\lim_{u\to 3} \dfrac{\ln\dfrac{u}{3}}{u-3} =$

ANSWER ONLY

(e) $\lim_{u\to-\infty} \left(\sqrt{u^2+2u+4}-\sqrt{u^2-3u+1}\right) =$

ANSWER ONLY

2. BRIEF SOLUTIONS

[3 MARKS EACH] For each of the following functions answer the question; if the object(s) requested does/do not exist, write "NONE".

(a) The horizontal asymptotes to the graph of $g(x) = 2\arctan x - 1$ are

ANSWER ONLY

(b) If f is defined by $f(x) = \left\{ \begin{array}{ll} \frac{1}{x^2-4} & \text{if } x \neq -2, 0, 2 \\ 6 & \text{if } x = 2 \\ 5 & \text{if } x = 0 \\ -4 & \text{if } x = -2 \end{array} \right\}$, the vertical asymptotes to the graph of f are

ANSWER ONLY

(c) Air is being pumped into a spherical balloon so that its volume increases at a rate of 10 cm^3/s. How fast is the radius of the balloon increasing when the radius is 12 cm?

ANSWER ONLY

3. BRIEF SOLUTIONS

[3 MARKS EACH] Evaluate each of the following, and *always simplify your answers as much as possible.*

(a) $\dfrac{d}{dx}\left(x^{\frac{x}{\ln x}}\right) =$

ANSWER ONLY

(b) $\dfrac{d}{du}\cos(\arcsin u) =$

ANSWER ONLY

(c) An antiderivative $F(x)$ of $f(x) = \sinh x$ such that $F(0) = -1$ is

ANSWER ONLY

(d) Where $f(t) = \dfrac{t^2}{1-t}\sqrt{\dfrac{3-t}{(3+t)^2}}$, $f'(2) =$

ANSWER ONLY

4. SHOW ALL YOUR WORK!

(a) [6 MARKS] Use Rolle's Theorem and the Intermediate Value Theorem to show that the curve $y = 1 + 2x + x^3 + 4x^5$ crosses the x-axis exactly once.

(b) [4 MARKS] Showing all your work, determine the value of the constant K that will make the following function continuous at $x = 0$:

$$f(x) = \begin{cases} \dfrac{Kx^2}{1 - \cos x} & \text{if } x > 0 \\ 8 & \text{if } x \leq 0 \end{cases}.$$

5. SHOW ALL YOUR WORK!

The equation $x^5 + x^2y + y^3 = 4y + 3$ defines y implicitly as a function of x near the point $(x, y) = (1, 2)$. Showing all your work

(a) [3 MARKS] determine the value of y' at $(x, y) = (1, 2)$;

(b) [3 MARKS] determine the value of y'' at $(x, y) = (1, 2)$; and

(c) [3 MARKS] estimate y when $x = 0.97$ by using the tangent line to the curve at the point $(x, y) = (1, 2)$.

6. SHOW ALL YOUR WORK!

[10 MARKS] The function f is defined by $f(x) = \begin{cases} \dfrac{60}{1+x^2} & \text{for } 0 \leq x \leq 2 \\ 20 - 4x & \text{for } 2 < x \leq 5 \end{cases}$.

A rectangle with sides parallel to the coordinate axes has one vertex at the origin, one on the positive x-axis, one on the positive y-axis; and the fourth on the graph of f. Showing all your work, use the calculus — no other method will be accepted — to determine the maximum area of such a rectangle.

7. SHOW ALL YOUR WORK!

For $x \geq 0$, define $f(x) = xe^{-2x^2}$.

(a) [3 MARKS] Showing all your work, determine the intervals of its domain where f is increasing, and the intervals where it is decreasing.

(b) [3 MARKS] Showing all your work, determine whether f has local extrema, and classify them, if any, as maxima or minima. You are expected to base your classification on tests studied in this course.

(c) [3 MARKS] Showing all your work, determine all inflection points for f.

(d) [1 MARK] Sketch the graph of f.

CONTINUATION PAGE FOR PROBLEM NUMBER

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