VERSION 1

McGILL UNIVERSITY
FACULTY OF SCIENCE
FINAL EXAMINATION

MATHEMATICS 140 2005 09 CALCULUS I

EXAMINER: Professor W. G. Brown
ASSOCIATE EXAMINER: Prof. A. Ivrii

DATE: Tuesday, December 06th, 2005
TIME: 14:00 – 17:00 hours

FAMILY NAME:
GIVEN NAMES:

STUDENT NUMBER:

INSTRUCTIONS

1. Fill in the above clearly.
2. DO NOT TEAR PAGES FROM THIS BOOK! All your writing — even rough work — must be handed in. You may do rough work anywhere in the booklet.
3. Calculators are not permitted.
4. The examination booklet consists of this cover, Pages 1 through 8 containing questions; and Pages 9, 10, and 11, which are blank. *Your neighbour's version of this test may not be the same as yours.*
5. There are two kinds of problems on this examination, each clearly marked as to its type.
   - Some of the questions on this paper require that you SHOW ALL YOUR WORK! Their solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
   - Some of the questions on this paper require only BRIEF SOLUTIONS ; for these you are expected to write the correct answer in the box provided; you are not asked to show your work, and you should not expect partial marks for solutions that are not completely correct.

   You are expected to simplify your answers wherever possible.

   You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)
6. A TOTAL OF 70 MARKS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a) /2	1(b) /2	1(c) /2	1(d) /2	1(e) /2	2(a) /2	2(b) /2	2(c) /2	2(d) /2
2(e) /2	3(a) /6	3(b) /4	4 /5	5 /5	6 /10		7(a) /1	7(b) /1
7(c) /2	7(d) /2	7(e) /2	7(f) /2	8(a) /5	8(b) /5			TOTAL /70

Closed Book.
Dictionaries are allowed.

1. BRIEF SOLUTIONS

[2 MARKS EACH] Give the numeric value of each of the following limits if it exists; if the limit is $+\infty$ or $-\infty$, write $+\infty$ or $-\infty$ respectively. In all other cases write "NO FINITE OR INFINITE LIMIT".

(a) $\lim_{y\to-\infty}\left(\sqrt{y^2+y}+y\right)=$

ANSWER ONLY

(b) $\lim_{x\to\infty}\left(e^{-x}\sinh x\right)=$

ANSWER ONLY

(c) $\lim_{x\to8}\dfrac{\sqrt{x+8}+\sqrt{2x}}{x^2+8x}=$

ANSWER ONLY

(d) $\lim_{x\to1}\left(\dfrac{1}{1-x}+\dfrac{1}{\ln x}\right)=$

ANSWER ONLY

(e) $\lim_{x\to\infty}\dfrac{\sin e^x}{\cos e^x}=$

ANSWER ONLY

2. BRIEF SOLUTIONS

[2 MARKS EACH] Evaluate each of the following, and simplify your answers as much as possible.

(a) $\dfrac{d}{dx}\left(\dfrac{x^2+3x}{x}\right) =$

ANSWER ONLY

(b) $\dfrac{d}{du}\left(u^u\right) =$

ANSWER ONLY

(c) An antiderivative $F(x)$ of $f(x) = 5x^4 + 2x^5$ such that $F(0) = 3$ is

ANSWER ONLY

(d) If $f(x) = x^3 + 7$, its inverse function $f^{-1}(x) =$

ANSWER ONLY

(e) $\dfrac{d}{dx}\left(|x|^4\right) =$

ANSWER ONLY

3. SHOW ALL YOUR WORK!

(a) [6 MARKS] Showing all your work, determine values of the constants a and b that will make the following function continuous everywhere.

$$f(x) = \begin{cases} (1+x)^{\frac{1}{x}} & \text{if } x > 0 \\ a + bx & \text{if } -1 \leq x \leq 0 \\ \dfrac{\sin(x+1)}{x+1} & \text{if } x < -1 \end{cases}$$

(b) [4 MARKS] Determine whether f is differentiable at $x = -1$. (For the purpose of this question you may assume that e is approximately 2.72.)

4. SHOW ALL YOUR WORK!

[5 MARKS] Let $f(x) = x^2 e^x$. Prove carefully by mathematical induction that

$$\frac{d^n f}{dx^n}(x) = \left(x^2 + 2nx + (n-1)n\right) \cdot e^x$$

for all positive integers n.

5. SHOW ALL YOUR WORK!

[5 MARKS] Let $g(x) = 2x - 3 + \cos x$. Use Rolle's Theorem or the Mean Value Theorem, to prove carefully that there exists exactly one real number x such that $g(x) = 0$. (π may be taken to be approximately 3.14.)

6. SHOW ALL YOUR WORK!

[10 MARKS] A rectangular poster is to be printed on a rectangular board of minimum area, leaving margins at the 4 sides. The top and bottom margins are each 10 cm, and the side margins are each 4 cm. If the printed area on the poster is fixed at 1,000 cm^2, find the best dimensions for the board. Show all of your work, and justify all of your statements. *In your solution, you are expected to carefully apply either the First or the Second Derivative Test, naming the test as you apply it.*

7. SHOW ALL YOUR WORK!

Let $f(x) = \sqrt{x^2 - 1}$.

(a) [1 MARK] State the domain of f.

(b) [1 MARK] State precisely where f is differentiable.

(c) [2 MARKS] Define when a line $x = a$ is a vertical asymptote to the graph of f.

(d) [2 MARKS] Either

i. Find all vertical asymptotes; or

ii. Explain why the graph has no vertical asymptotes.

(e) [2 MARKS] Determine the global maximum value of f, or explain why there is none.

(f) [2 MARKS] Determine the global minimum value of f, or explain why there is none.

8. SHOW ALL YOUR WORK!

A function $f(t)$ satisfies, for all real numbers t, the equation

$$t^3 + f(t)^3 + 6t^2 \cdot f(t) = 8\,.$$

(a) [5 MARKS] Find an equation for the tangent to the graph $y = f(t)$ at the point $(t, y) = (2, 0)$.

(b) [5 MARKS] Showing all your work, determine the value of $f''(2)$.

CONTINUATION PAGE FOR PROBLEM NUMBER

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