VERSION 1

McGILL UNIVERSITY
FACULTY OF SCIENCE
FINAL EXAMINATION

MATHEMATICS 140 2004 09 — CALCULUS I

EXAMINER: Professor W. G. Brown
ASSOCIATE EXAMINER: Prof. N. Sancho

DATE: Thursday, December 09th, 2004
TIME: 09:00 - 12:00 hours

FAMILY NAME:
GIVEN NAMES:

STUDENT NUMBER:

INSTRUCTIONS

1. Fill in the above clearly.
2. DO NOT TEAR PAGES FROM THIS BOOK! All your writing — even rough work — must be handed in. You may do rough work anywhere in the booklet.
3. Calculators are not permitted.
4. The examination booklet consists of this cover, Pages 1 through 8 containing questions; and Pages 9, 10, and 11, which are blank. *Your neighbour's version of this test may not be the same as yours.*
5. There are two kinds of problems on this examination, each clearly marked as to its type.
   - Some of the questions on this paper require that you SHOW ALL YOUR WORK! Their solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
   - Some of the questions on this paper require only BRIEF SOLUTIONS; for these you are expected to write the correct answer in the box provided; you are not asked to show your work, and you should not expect partial marks for solutions that are not completely correct.

   You are expected to simplify your answers wherever possible.

   You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)
6. A TOTAL OF 100 MARKS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a) /3	1(b) /3	1(c) /3	1(d) /3	1(e) /3	1(f) /3	2(a) /3	2(b) /3	2(c) /3
2(d) /3	2(e) /3	3(a) /5	3(b) /5	3(c) /5	4(a) /6	4(b) /6	5(a) /4	5(b) /8
6 /10	7(a) /1	7(b) /4	7(c) /4	7(d) /3	8 /6			TOTAL /100

1. BRIEF SOLUTIONS

[3 MARKS EACH] Give the numeric value of each of the following limits if it exists; if the limit is $+\infty$ or $-\infty$, write $+\infty$ or $-\infty$ respectively. In all other cases write "NO FINITE OR INFINITE LIMIT".

(a) $\lim_{y\to-\infty} \frac{|y|}{y} =$

ANSWER ONLY

(b) $\lim_{u\to-\infty} \frac{\sin u}{u} =$

ANSWER ONLY

(c) $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{2n} =$

ANSWER ONLY

(d) $\lim_{x\to0} \left(\frac{1}{x} - \frac{1}{\sin x}\right) =$

ANSWER ONLY

(e) $\lim_{t\to\infty} \left(\ln(3t^2) - \ln(t^2+7)\right) =$

ANSWER ONLY

(f) $\lim_{x\to8} \frac{\sqrt{x+8}-\sqrt{2x}}{x^2-8x} =$

ANSWER ONLY

2. BRIEF SOLUTIONS

[3 MARKS EACH] Determine each of the following derivatives, and simplify your answers as much as possible.

(a) $\dfrac{d}{dx}\left(\dfrac{x^2+3x}{x}\right) =$

ANSWER ONLY

(b) $\dfrac{d}{dt}\left(t^{-3t}\right) =$

ANSWER ONLY

(c) $\dfrac{d}{ds}\left(\tan(e^{2s}) - e^{2\tan s}\right) =$

ANSWER ONLY

(d) $\dfrac{d}{dy}\left(\cosh^2(3y)\right) =$

ANSWER ONLY

(e) $\dfrac{d}{dX}\left(\cos^2 X - \cos(X^2)\right) =$

ANSWER ONLY

3. BRIEF SOLUTIONS

(a) [5 MARKS] Find an equation for a line through the point $(-2, 0)$ which is tangent to the curve $y = x^2$ and is not horizontal.

ANSWER ONLY

(b) [5 MARKS] Determine values of the constants a and b that will make the following function continuous at $x = -6$.

$$f(x) = \begin{cases} \sqrt{x+31} & \text{if } x < -6 \\ a + b & \text{if } x = -6 \\ a(x+5) & \text{if } x > -6 \end{cases}$$

ANSWER ONLY

(c) [5 MARKS] Determine values of the constants k and ℓ that will make the following function differentiable at $x = 1$.

$$g(x) = \begin{cases} kx^2 + \ell & \text{if } x \leq 1 \\ 6x - 4 & \text{if } x > 1 \end{cases}$$

ANSWER ONLY

4. SHOW ALL YOUR WORK!

(a) [6 MARKS] Coffee is draining from a conical filter of depth 10 cm and diameter 10 cm (at the top) into a cylindrical coffee pot of diameter 12 cm, at the rate of 100 cm^3/min. Determine how fast, in cm/min, the level of coffee in the pot is rising when the coffee in the filter is 3 cm deep?

(b) [6 MARKS] You are given that $y = y(t)$ is a function of t satisfying $t^3y + ty^3 = 2$. Assuming that $y(1) = 1$, determine the values of $y'(1)$ and $y''(1)$.

5. SHOW ALL YOUR WORK!

(a) [4 MARKS] Prove that the function $x^3 + 9x^2 + 33x$ assumes the value -8 at least once.

(b) [8 MARKS] Using the Mean Value Theorem or Rolle's Theorem — no other methods will be accepted — prove carefully that $x^3 + 9x^2 + 33x$ takes on the value -8 *at most* once.

6. SHOW ALL YOUR WORK!

[10 MARKS] Using the calculus carefully, determine how to express 8 as the sum of 2 nonnegative real numbers such that the sum of the square of the first and the cube of the second is as small as possible.

7. SHOW ALL YOUR WORK!

Let $f(x) = \left(\dfrac{x}{x+3}\right)^2$.

(a) [1 MARKS] State the domain of f.

(b) [4 MARKS] Find the intervals of increase and the intervals of decrease of f.

(c) [4 MARKS] Determine the intervals of concavity upwards and the intervals of concavity downwards, and the inflection points, if any.

(d) [3 MARKS] Sketch the graph of $y = f(x)$, showing — clearly labelled — all horizontal and all vertical asymptotes.

8. SHOW ALL YOUR WORK!

[6 MARKS] Consider the function $f(x) = 5x + 9$ near $x = -1$. Is it, or is it not true that f is continuous at $x = -1$? If the statement is true, prove it carefully, using the ϵ-δ definition. If it is false, prove that carefully.