NOTE TO PRINTER

(These instructions are for the printer. They should not be duplicated.)

THIS EXAMINATION SHOULD BE PRINTED ON $8\frac{1}{2} \times 14$ PAPER, AND STAPLED WITH 3 SIDE STAPLES, SO THAT IT OPENS LIKE A LONG BOOK.

McGILL UNIVERSITY
FACULTY OF SCIENCE
FINAL EXAMINATION

MATHEMATICS 189–140A

CALCULUS I

EXAMINER: Professor W. G. Brown — DATE: Friday, December 10, 1999
ASSOCIATE EXAMINER: Professor D. Sussman — TIME: 14:00 – 17:00 hours

FAMILY NAME:

MR, MISS, MS, MRS, &c.:

GIVEN NAMES:

STUDENT NUMBER:

Instructions

1. Fill in the above clearly.
2. Do not tear pages from this book; all your writing — even rough work — must be handed in.
3. Calculators are not permitted.
4. This examination booklet consists of this cover, Pages 1 through 10 containing questions; and Pages 11 and 12, which are blank. You are expected to show all your work. All solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
5. You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a)	1(b)	2	3	4(a)	4(b)	4(c)	5(a)	5(b)
/6	/3	/8	/8	/1	/5	/2	/3	/3
5(c)	5(d)	5(e)	5(f)	6	7	8(a)	8(b)	9
/3	/3	/2	/4	/8	/8	/8	/8	/8
10						Total	Term	
/9						/100	/10	

1. Let $f(x) = \begin{cases} \dfrac{x + \sin 3x}{\tan 4x} & x > 0 \\ \dfrac{(x-a)^2}{4} & x \leq 0 \end{cases}$, where a is a constant, to be determined.

(a) [6 MARKS] Determine each of $\lim_{x \to 0^-} f(x)$, $\lim_{x \to 0+} f(x)$ or explain why either or both do not exist.

(b) [3 MARKS] Use the information of part (a) to determine all values, if any, for the constant a, which will make f continuous at $x = 0$.

2. [8 MARKS] Let $m(x) = x^{x^2} - (x^2)^{-3}$. Determine the value of $m'(1)$.

3. [8 MARKS] If y is defined implicitly as a function of x by $2x^2 - 3xy + 5y^2 = 10$, determine the value of $\dfrac{d^2y}{dx^2}$ when $(x, y) = (1, -1)$.

4. For the function $h(x) = \arctan \dfrac{2+x}{1-2x}$,

(a) [1 MARK] State the (largest possible) domain.

(b) [5 MARKS] Determine $\dfrac{dh}{dx}$ for all points x where the derivative exists.

(c) [2 MARKS] Give an example of a function different from h which has exactly the same domain and exactly the same derivative as h.

5. The function u is defined by $u(x) = \dfrac{x}{1+x^2}$, for $-\infty < x < \infty$.

(a) [3 MARKS] Determine the intervals where the function u is increasing, and those where it is decreasing.

(b) [3 MARKS] Find all critical points. In each case determine whether the point is a maximum or minimum point, or neither.

(c) [3 MARKS] Determine the intervals where the graph of u is concave upwards, and those where it is concave downwards.

(d) [3 MARKS] Determine all inflection points of the graph.

(e) [2 MARKS] Determine all horizontal or vertical asymptotes of the graph.

(f) [4 MARKS] Sketch the graph.

You may assume that $u'(x) = -\dfrac{(x-1)(x+1)}{(x^2+1)^2}$, and that $u''(x) = \dfrac{2x(x^2-3)}{(x^2+1)^3}$. (For each of parts (a) through (e) you are expected to show all your work and your results, clearly marked by the question number, e.g., 5(c); it is not sufficient to provide information only on your graph.)

6. [8 MARKS] Use the Intermediate Value Theorem and/or the Mean Value Theorem and/or properties of $G'(x)$ to show that the function $G(x) = x^2 - e^{\frac{1}{1+x}}$ assumes the value 0 for exactly one real number x such that $0 < x < 2$. Show all your work. [Hint: You may assume that $e^{\frac{1}{3}} < 2$.]

7. [8 MARKS] Triangle OBC, in the first quadrant, has vertex O at the origin, vertex B on the x-axis, and vertex C on the y-axis. If the vertices are constrained so that the line joining B and C passes through the point $(2, 3)$, determine the minimum area for triangle OBC. Show all your work.

8. Showing all your work, evaluate the following limits, if they exist:

(a) [8 MARKS] $\lim_{x\to\infty} (\sqrt{x^2+x} - \sqrt{x^2-x})$.

(b) [8 MARKS] $\lim_{x\to 0} \dfrac{\tan x - \sin x}{x^3}$.

9. [8 MARKS] Showing all your work, determine all lines with slope 3 which are normal to the curve $64y + x^3 = 0$, (i.e. which are perpendicular to the tangent at each point where they meet the curve).

10. [9 MARKS] Showing all your work, determine the (global) maxima and minima of the function $R(x) = 3x^4 + 4x^3 - 6x^2 - 12x$ on the closed interval $-2 \le x \le 2$. [Hint: $x^3 + x^2 - x - 1 = (x^2 - 1)(x + 1)$.]

CONTINUATION PAGE FOR PROBLEM NUMBER $\boxed{}$

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