1. (20 points) Find the limit if it exists, showing your main calculations. If the limit doesn't exist, explain why.

   (a) $\lim_{x\to+\infty} \dfrac{\sin(3x^2)}{x+1}$

   (b) $\lim_{x\to 3} \dfrac{\sqrt{x+6}-x}{|x-3|}$

   (c) $\lim_{x\to 0} \dfrac{e^{3x}-e^{-2x}}{1-\cos(2x)}$

   (d) $\lim_{x\to\infty} \dfrac{3x^2-\ln(x)}{4x^2+8x+1}$

2. (35 points) For the function $f(x) = \dfrac{x^3}{x^2-1}$ determine where the function f:

   (a) is increasing

   (b) is decreasing

   (c) is concave up

   (d) is concave down

   (e) has local maxima

   (f) has local minima

   (g) has inflection points.

   Also:

   (h) Write down the equation of the asymptotes of the graph of the function, if any.

   (i) Draw the graph of this function.

   (All questions prior to the last one should be answered before drawing the graph. Just incorporating your answer into your sketch of the graph without writing the answers to these questions is not acceptable.)

3. (20 points)

   (a) Find the ratio of the height to the radius of the base of the circular cylinder with minimal total surface area (top, bottom and the side) among all the circular cylinders of a fixed volume.

   (b) Find the ratio of the height to the radius of the base of the open-topped circular cylinder with minimal area (side and bottom) among all circular cylinders of a fixed volume.

4. (20 points)

   (a) In the following, find $\dfrac{dy}{dx}$:

   i. $y = e^{x^2} + \tan^{-1}\left(\dfrac{1}{x+2}\right)$

   ii. $\sin^{-1}(xy) + \ln\left(\dfrac{x}{y}\right) = y$

   (b) In the equation $y^2 + xy = 20$, find $\dfrac{d^2y}{dx^2}$ (second derivative) only in terms of x and y.

5. (15 points) Consider the curve $x^3 + y^3 - 3xy = 3$. Verify that $(1, 2)$ is a point on this curve and find the equation of the tangent line at this point.

6. (10 points) Show that the equation $x^3 - 3x + 1 = 0$ has exactly one solution in the interval $[-1, 1]$.

7. (10 points) Which values of a, if any at all, make the following function continuous everywhere? Explain.

$$f(x) = \begin{cases} 4a - x^2 & \text{if x} < 0 \\ a(x-a)^2 & \text{if } 0 \leq \text{x} \leq 1 \\ -6(x-a) & \text{if } 1 < \text{x} \end{cases}$$

8. (10 points) Using differentials, find a good approximation to $\sqrt[5]{31}$.

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-139B

CALCULUS

Examiner: A. Rajaei
Associate Examiner: Professor W.G. Brown

Date: Thursday, April 13, 2000
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Calculators are not permitted.

This exam comprises the cover and two pages of questions.