FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS MATH139

Calculus

Examiner: Professor S. W. Drury — Date: Thursday, 13 December 2007

Associate Examiner: Professor W. Brown — Time: 2: 00 pm. – 5: 00 pm.

INSTRUCTIONS

Answer all questions.
You are expected to simplify your answers wherever possible.
This is a closed book examination.
Calculators are not permitted.

Questions 1 thru 3 are worth 9 points each, questions 4 thru 9 are worth 10 points each.
The exam will be marked out of 87 points and then scaled to a percentage.

This exam has 9 questions and 3 pages

1. (9 points) Find the following limits. You may use L'Hospital's Rule, when appropriate, if you wish.

$$\text{(a) } \lim_{x\to\frac{\pi}{4}} \frac{\cos(2x)}{\tan(x)} \qquad \text{(b) } \lim_{x\to\infty} \frac{\ln(3x^4+1)}{\ln(4x^5+2)}$$

$$\text{(c) } \lim_{x\to 3} \frac{x^2-x-6}{\sin(\pi x)}$$

2. (9 points) Find the derivative of each of the following functions

$$\text{(a) } f(x) = 2x^4 - 3x^{-5} \qquad \text{(b) } f(x) = \frac{x^2}{2x^3+1} \qquad \text{(c) } f(x) = x\ln(x^2+1)$$

3. (9 points) Find the derivative of each of the following functions

$$\text{(a) } f(x) = \arcsin(x^{-\frac{1}{3}}) \qquad \text{(b) } f(x) = xe^{x^{-1}} \qquad \text{(c) } f(x) = \sqrt{\frac{\cosh(x)}{(2+x^2)^3}}$$

4. (10 points) Find all horizontal and vertical asymptotes of the graph of

$$f(x) = \frac{5e^x+4}{e^x-2}$$

For each asymptote that you have found, justify your answer by writing down a limit which implies the existence of the asymptote.

5. (10 points) Find the equation of the line tangent to the curve

$$x^5 + y^3\ln(x) + y^2 = 2$$

at the point $(x, y) = (1, 1)$.

6. (i) (3 points) Find all critical points of the function $f(x) = 2x\cos(x) + (x^2 - 2)\sin(x)$ in the interval $-\pi \leq x \leq 2\pi$.

(ii) (4 points) Classify each such point as a local minimum, a local maximum or some other kind of critical point.

(iii) (3 points) Find the absolute minimum value of the function $x \mapsto f(x)$ on the interval $-\pi \leq x \leq 2\pi$.

7. (i) (4 points) Find the first derivative and second derivative of the function

$$f(x) = (16 - 11x + 2x^2)e^x.$$

(ii) (3 points) Determine where the function is increasing and decreasing.

(iii) (3 points) Determine where the function is concave up and concave down.

8. (10 points) A Ferris wheel has a diameter of 20 meters and is rotating at a rate of one revolution per minute. How fast (in meters per second) is a person descending when they are 6 meters away (horizontally) from the vertical plane through the axis of the wheel.

9. (10 points) A propane tank is made of a cylindical part capped by two hemispheres. The hemispheres are twice as expensive per unit area as the cylindrical wall. It is desired to make a tank with a fixed volume (say V cubic units where V is a constant) as economically as possible. Find the ratio of the total length of the tank to the radius of the cylindrical part in the most economical configuration. You are given the following formulæ. The volume of a sphere is $\frac{4}{3}\pi r^3$ cubic units and its area is $4\pi r^2$ square units where the radius is r linear units. The circumference of a circle of radius r linear units is $2\pi r$ linear units, and its area is πr^2 square units.

$\star$ $\star$ $\star$ $\star$ $\star$