1. (9 marks) Find the following limits. You may use L'Hospital's Rule, when appropriate, if you wish.

$$\text{(a)} \lim_{x\to -3} \frac{\cos(\pi x)}{x^2+1} \qquad \text{(b)} \lim_{x\to -\infty} \frac{5e^{3x}+2|x|}{8e^{3x}+x}$$

$$\text{(c)} \lim_{x\to -3} \frac{\sin(\pi x)}{x^2+4x+3}$$

2. (9 marks) Find the derivative of each of the following functions

$$\text{(a) } f(x) = 2x^{-3} - x^{-5} \qquad \text{(b) } f(x) = \frac{x^5}{3x^2+1} \qquad \text{(c) } f(x) = x^2\ln(x^3+5)$$

3. (9 marks) Find the derivative of each of the following functions

$$\text{(a) } f(x) = \arcsin(x^{-2}) \qquad \text{(b) } f(x) = e^{x^2} \qquad \text{(c) } f(x) = \ln(1+(\cos x)^2)$$

4. (10 marks) Find all horizontal and vertical asymptotes of the graph of

$$f(x) = \frac{\sqrt{x^4+5}}{x^2-6x+8}.$$

For each asymptote that you have found, justify your answer by writing down a limit which implies the existence of the asymptote.

5. (10 marks) Find the equation of the line tangent to the curve

$$3x^5 + xy^3 + 2y^2 = 4$$

at the point $(x, y) = (1, -1)$.

6. (i) (3 marks) Find all critical points of the function $f(x) = \cos(x) + (x - \pi)\sin(x)$ in the interval $0 \le x \le 3\pi$.
   (ii) (4 marks) Classify each such point as a local minimum, a local maximum or some other kind of critical point.
   (iii) (3 marks) Find the absolute maximum value of the function $x \mapsto f(x)$ on the interval $0 \le x \le 3\pi$.

7. (i) (4 marks) Find the first derivative and second derivative of the function

$$f(x) = (x^3 - x^2 + 2x - 2)e^x.$$

   (ii) (3 marks) Determine where the function is increasing and decreasing.
   (iii) (3 marks) Determine where the function is concave up and concave down.

8. (10 marks) When a spherical tank of radius a feet is filled with water to a depth of x feet, the volume of water that it contains is $\frac{1}{3}\pi x^2(3a - x)$ cubic feet (for $0 \le x \le 2a$). If the tank has radius 5 feet and is being drained at a constant rate of 3 cubic feet per minute, how fast is the water level declining when the depth of water in the tank is 2 feet. Give your answer in feet per minute.

9. (10 marks) A lamp emitting light equally in all directions is mounted on a vertical track above a horizontal bench. The degree of illumination of a tiny piece of paper lying flat on the bench is proportional to $r^{-2}\sin(\theta)$ where r is the distance between the lamp and the piece of paper and θ is the angle between the path of the light and the surface of the bench. If the piece of paper is at a horizontal distance of one metre from the track, how high above the bench should the light be positioned in order to illuminate the paper to the greatest degree. You should assume that the only light to reach the paper comes directly from the lamp.

⋆ ⋆ ⋆ ⋆ ⋆

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS MATH139

Calculus I

Examiner: Professor S. W. Drury
Associate Examiner: Professor W. Brown

Date: Friday, 5 December 2003
Time: 9: 00 am. – 12: 00 noon

INSTRUCTIONS

Another calculus exam is being written in the same building.
This is the exam for MATH139.
Please make sure that you have the correct exam paper.

Answer all questions.
This is a closed book examination.
Calculators are not permitted.

Questions 1 thru 3 are worth 9 points each, questions 4 thru 9 are worth 10 points each.
The exam will be marked out of 87 points and then scaled to a percentage.

This exam comprises the cover and 2 pages of questions.