1. (9 marks) Find the following limits. You may use L'Hospital's Rule if you wish.

$$\text{(a) } \lim_{x\to 3} \frac{1}{x-3} - \frac{6}{x^2-9} \qquad \text{(b) } \lim_{x\to\infty} \frac{3e^{2x}+4}{2e^{2x}+5}$$

$$\text{(c) } \lim_{x\to 0} \frac{\arctan x}{x}$$

2. (9 marks) Find the derivative of each of the following functions

$$\text{(a) } f(x) = \sqrt{4x+3} \qquad \text{(b) } f(x) = \frac{4x}{x^2+5} \qquad \text{(c) } f(x) = (\ln x)^6$$

3. (9 marks) Find the derivative of each of the following functions

$$\text{(a) } f(x) = \ln(e^x+1) \qquad \text{(b) } f(x) = 5^x \qquad \text{(c) } f(x) = \sin x \cos x$$

4. (10 marks) Find the equation of the line tangent to the curve

$$\sqrt{2x+7y} + \sqrt{6xy} = 11$$

at the point $(x, y) = (2, 3)$.

5. (i) (4 marks) Find all the critical points of the function $f(x) = \cos x + x \sin x$ in the interval $-\frac{\pi}{2} < x < \frac{5\pi}{2}$.
   (ii) (3 marks) Classify each such point as a local minimum, a local maximum or some other kind of critical point.
   (iii) (3 marks) Find the absolute maximum and absolute minimum values of the function $x \mapsto f(x)$ on the interval $-\frac{\pi}{2} \le x \le \frac{5\pi}{2}$.

6. (i) (4 marks) Find the first derivative and second derivative of the function

$$f(x) = (2x^2 - 9x + 11)e^x.$$

   (ii) (3 marks) Determine where the function is increasing and decreasing.
   (iii) (3 marks) Determine where the function is concave up and concave down.

7. (10 marks) Find the point on the line $2x + y = 3$ nearest to the point $(5, 8)$. (*Hint :* In minimizing a distance, it may be easier to minimize the square of the distance.)

8. (10 marks) A piece of wire 1 metre long is bent into the shape of the perimeter of a sector of a circle (so that the wire occupies two radii and an arc of the circle). Find the angle θ at the apex of the sector that maximizes the area of the sector. (*Hint :* The area of a sector is given by the formula $\frac{1}{2}r^2\theta$ where the radius is r metres and θ is the angle at the apex, i.e. between the two radii.)

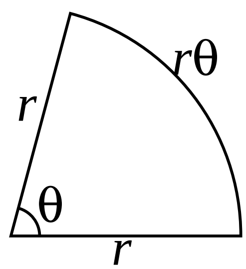

Figure for question 8.

9. A function f is defined on the whole real line by

$$f(x) = \begin{cases} x & \text{if } -\pi < x < 0, \\ \sin x & \text{otherwise.} \end{cases}$$

(i) (2 marks) Is f continuous at $x = -\pi$? If not, what kind of discontinuity does f have at $-\pi$?

(ii) (2 marks) Is f continuous at $x = 0$? If not, what kind of discontinuity does f have at 0?

(iii) (4 marks) Find $\lim_{x \to 0-} \frac{f(x) - f(0)}{x - 0}$ and $\lim_{x \to 0+} \frac{f(x) - f(0)}{x - 0}$.

(iv) (2 marks) Is f differentiable at $x = 0$?

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FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-139A

Calculus I

Examiner: Professor S. W. Drury
Associate Examiner: Professor W. Brown

Date: Friday, 8 December 2000
Time: 9: 00 am. – noon

INSTRUCTIONS

Answer all questions.
This is a closed book examination.
Calculators are not permitted.

Questions 1 thru 3 are worth 9 points each, questions 4 thru 9 are worth 10 points each.
The exam will be marked out of 87 points and then scaled to a percentage.

This exam comprises the cover and 2 pages of questions.