

1. A jar has 3 blue and 2 white balls; a second jar has 2 blue and 5 white balls. A single fair die is rolled and, if a 1 or 2 comes up, a ball is drawn out of the second jar; otherwise a ball is drawn out of the first jar. If the ball drawn is blue, what is the probability that it came out of the first jar? Out of the second jar?

2. Evaluate the following integrals

$$(a) \int \frac{(\ln x)^2}{x} dx \quad (b) \int xe^{2x} dx, \quad (c) \int \frac{e^{2x}}{1+e^x} dx.$$

3. Find the area bounded by $f(x) = x^2 - x$ and $g(x) = 2x$ for $-2 \leq x \leq 3$. (Hint: Sketch the two curves and find where they intersect.)

4. Estimate the area of the region under the graph bounded by $y = 1 + 0.5x^2$, x -axis, $x = 0$ and $x = 1$; by dividing into four rectangles of equal width.

(a) Use the left-end point rule, (b) use the right-end point rule.

5. Find the equilibrium price and then the consumers' surplus and producers' surplus of the equilibrium price level if

$$p = D(x) = 20 - 3x \text{ and } p = S(x) = 2 + x^2.$$

6. Find the critical points of the function

$$f(x, y) = x^3 - 3xy + y^3 - 1,$$

then use the second derivative test to classify the nature of each point. Finally determine the relative extrema of the function.

7. Postal regulations specify that the combined length and girth of a parcel sent by parcel post may not exceed 108 inches. Find the dimensions of the rectangular package (width x , length y and height z) that would have the greatest possible volume under these regulations. (Girth + length = $2x + 2z + y$ and volume $V = xyz$.)

8. Using the method of Lagrange multiplier find the maximum and minimum of the function $f(x, y) = xy$ subject to the constraint $x^2 + y^2 = 16$.
9. Evaluate $I = \int_0^1 dy \int_y^1 e^{-x^2} dx$ by reversing the order of integration. (Hint: Sketch the region of integration in the xy -plane.)
10. The joint probability density function for x and y is given by

$$f(x, y) = \begin{cases} c(x + y) & , 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & , \text{elsewhere.} \end{cases}$$

- (a) Show that $c = 1$. (b) Find $P(x + y \leq 1)$.

McGILL UNIVERSITY
FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-131B

MATHEMATICS FOR MANAGEMENT II

Examiner: Professor N.G.F. Sancho
Associate Examiner: Professor C. Roth

Date: Tuesday, April 25, 2000
Time: 9:00 A.M. - 12:00 Noon.

INSTRUCTIONS

Non-programmable calculators are permitted.

This exam comprises the cover and two pages of questions.