NAME: _____   ID #: _____

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 122

CALCULUS FOR MANAGEMENT

INSTRUCTIONS

1. Please answer questions in space provided.

2. This is a closed book exam.

3. Non-graphing, non-programmable calculators are permitted.

4. Translation dictionaries are permitted.

This exam is comprised of the cover page and 8 pages of questions (for a total of 9 questions) and a blank page for your work.

1. (5 marks) If \$1000 is deposited in an account yielding 7% interest rate compounded semi-annually, write a formula for the amount accumulated after 4 years. How long does it take for the money to double? Simplify your answers as much as possible without giving the actual numerical answer.

2. (5 marks) Determine the value of k that will make the following function continuous: $f(x) = \begin{cases} 2x^2 & x < 1 \\ kx^3 + (1+k) & x \geq 1 \end{cases}$

3. (5 marks each) Find the derivatives of the following functions. Please simplify completely.

   (a) $f(x) = 5xe^{2x}$

   (b) $f(x) = \dfrac{\ln(2 + x^2)}{x}$

(c) $f(x) = \sqrt{(x^3 + 2)(x^2 - 1)^2}$

4. (5 marks each) Evaluate the following:

   (a) $\int (3x + 2)(3x^2 + 4x + 2)^{100} \, dx$

(b) $\displaystyle\int x \ln x \, dx$

(c) $\displaystyle\int_0^\infty \frac{1}{(x+1)^2} \, dx$

5. (5 marks) Find the equation of the tangent line at $(1,1)$ on the curve $x + \ln y = x^2 y^3$

6. (5 marks) Find the area bounded by the two curves $y = x^2 + 1$ and $y = -2x + 4$.

7. (10 marks) Suppose the supply function for a certain item is given by $S(q) = q^2 + 10q$ and the demand equation is given by $D(q) = 900 - 20q - q^2$. Find the consumers' surplus and the producers' surplus. (note that $(45)^2 = 2025$)

8. (10 marks) Tesla Motors builds electric cars in California. They have found that building one car per week costs them $ 97 500, and that building two cars per week costs them a total of $ 144 500. The demand for the cars has been found to be $p = -x + 47000 + \dfrac{300000}{x} - \dfrac{54}{x^2}$. Assuming the cost function to be linear, find how many cars per week Tesla Motors must sell in order to maximize their profit. Justify your answer completely.

9. (20 marks) Graph $y = \dfrac{1}{x^2 - 4}$, considering the domain, critical points, regions where the function is increasing and decreasing, points of inflection, regions where the function is concave up and concave down, intercepts and asymptotes.

Final Exam Math 122 Monday December 14, 2009 2-5pm.

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