Name:_____         Student number:_____

# McGILL UNIVERSITY

# FACULTY OF SCIENCE

# FINAL EXAMINATION

# MATH 122

# CALCULUS FOR MANAGEMENT

Examiner: Dr. A. Hundemer                    Date: Thursday December 6, 2007
Associate Examiners: E. Kritchevski & M. Pinsonnault  Time: 9:00 AM - 12:00 PM

# INSTRUCTIONS

1. Please answer all questions in the exam booklets provided .

2. This is a closed book exam.

3. Calculators are not permitted.

4. Dictionaries are not permitted.

5. This exam must be returned with the exam booklet.

This exam comprises the cover page, and 10 questions

### McGill University
### MATH122 Final Examination
### December 6 2007, 9:00-12:00 am

*You have **3 hours** to complete this test. Simplify your answers as much as possible without giving the actual numerical value. Marks for each question are indicated; there is a total of 100. Please put away your calculator and turn off your cell phone; there are no aids allowed for this test.*

**Q1.** Suppose $P$ dollars are deposited in an account with a yearly interest rate of $r\%$, compounded $m$ times per year.

    **a.** (4 marks) Write down a formula for the **balance** $A$ after $t$ years.

    **b.** (4 marks) Find the rate of change of $A$ with respect to $r$.

    **c.** (4 marks) With an interest rate of 5% per year, compounded once a year, how long will it take for the balance to **double**?

**Q2.** (4 marks) Assuming **continuous** compounding, with an interest rate of 6% per year, what would be the value of an initial investment of 1 dollar after 5 years?

**Q3.** (4 marks) You created a new blog on financial mathematics. Two days after you put it online, 2 people registered. One day later, the number of registered members had risen to 5. How many days from the opening would it take for the number of registered members to reach 1 million if we assume an **exponential growth** on $[2, \infty)$ ?

**Q4.** (4 marks) We define a function $f(x)$ by setting

$$f(x) = \begin{cases} kx^2, & \text{if } x \leq 2 \\ x + k, & \text{if } x > 2 \end{cases}$$

Find the value of the constant $k$ that makes $f(x)$ continuous.

**Q5.** The cost and demand (price) functions for a product are respectively $C(x) = x^2 - 2x + 50$ and $p(x) = 20(1 + 1/x^2)$, where $x$ is the number of items.

    **a.** (4 marks) In this context, what are the admissible values for $x$?

    **b.** (4 marks) Write down a formula for the marginal cost.

    **c.** (4 marks) Write down a formula for the marginal revenue.

    **d.** (4 marks) Write down a formula for the marginal profit.

    **e.** (4 marks) For what **admissible** value of $x$ is the profit maximal ? (This one is trickier. At some point you should get a degree 3 polynomial divisible by $(x - 1)$.)

**Q6.** Let $f(x) = x^4 - 8x^3 + 18x^2$.

    **a.** (4 marks) Find the $y$-intercept and the roots of $f(x)$ if any.

    **b.** (4 marks) Find the horizontal and vertical asymptotes if any.

    **c.** (4 marks) Find the critical numbers of $f(x)$.

    **d.** (4 marks) Determine where $f(x)$ is increasing or decreasing and where $f(x)$ has local maxima or minima.

    **e.** (4 marks) Determine where the graph of $f(x)$ is concave up or down and locate the inflection points.

**Q7.** Suppose the demand function (relating the demand $q$ and the price $p$ for some product) is of the form $q = Cp^{-k}$ where $C$ and $k$ are positive constants.

    **a.** (4 marks) Find the elasticity $E = -\frac{p}{q}\frac{dq}{dp}$.

    **b.** (4 marks) If $0 < k < 1$, which value of $p$ in $[1, 10]$ maximizes the revenue ?

    **c.** (4 marks) If $1 < k$, which value of $p$ in $[1, 10]$ maximizes the revenue ?

**Q8.** Using integration by parts, evaluate

    **a.** (4 marks) $\int_0^1 xe^{2x}\, dx$.

    **b.** (4 marks) $\int_1^2 x\ln(x)\, dx$.

**Q9.** Using substitution, find

    **a.** (4 marks) $\int (3x^2 - x)e^{(x^3 - \frac{x^2}{2})}\, dx$.

    **b.** (4 marks) $\int \frac{4x-6}{x^2-3x}\, dx$.

**Q10.** Suppose that the marginal revenue (in dollars per item) from a product is given by $f(x) = 50 - 3x - x^2$, where $x$ is the number of items.

    **a.** (4 marks) Find the revenue function for the product.

    **b.** (4 marks) Find the demand function for the product.

Total Marks $= (100)$