Evaluate the following integrals:

1. $\displaystyle\int \frac{\tan^2 x - 5\tan x + 9}{\tan^2 x - 5\tan x + 6}\sec^2 x dx.$

2. $\displaystyle\int \frac{(x-2)dx}{\sqrt{x+2}+2}.$

3. $\displaystyle\int x^2 \arcsin x dx.$

4. Find $f(1)$ if $\displaystyle\int_{e^x}^{e^{2x}} t^2 f(t)dt = (x+1)\ln(x+1).$

5. Given the curve $r = 2(1+\sin\theta)$:
   (a) Sketch the curve.
   (b) Find the area of the inner loop.
   (c) Write the length of the curve in the form of an interval.
   (d) Write the tangent line to the curve at $\theta = \dfrac{\pi}{2}$.

6. Sketch the region bounded above by $y = e^x$, below by the x-axis and to the right by $x = 1$. Find the volume of the figure obtained by revolving this region about the line $y = -2$.

Consider the following series:

7. $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{k^k}$, 8. $\displaystyle\sum_{k=1}^{\infty} (\sqrt{k^2+k} - k)^k$, 9. $\displaystyle\sum_{k=0}^{\infty} \frac{\cos \pi k}{\sqrt{k+1}}$.

Investigate each of them for convergence, absolute convergence (where appropriate) and conditional convergence (where appropriate).

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-121A

CALCULUS II

Examiner: Professor B. Lawruk
Associate Examiner: Professor W. Moser

Date: Tuesday, December 17, 1996
Time: 9:00 A.M. - 12:00 Noon

INSTRUCTIONS

Each question is worth 10 points

This exam comprises the cover and 1 page of questions.