McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 112

FUNDAMENTALS OF MATHEMATICS

Examiner: Professor C. Roth   Date: Monday December 17, 2007
Associate Examiner: Professor A. Hundemer   Time: 2:00 PM-5:00 PM

INSTRUCTIONS

Please answer questions in the exam booklets provided.
This is a closed book exam.
Non-programable calculators are permitted and required.
Use of a regular and or translation dictionary is permitted.
This exam is a total of 105 marks.

This exam comprises the cover page, and three pages of 16 questions.

1. (5 Marks) A group of friends decides to buy a vacation home for $ 120,000, sharing the cost equally. If they can find one more person to join them, each person's contribution will drop by $ 6000. How many people are in the group?

2. (5 Marks) Find values of k so that the lines $3kx + 8y = 5$ and $6y - 4kx = -1$ will be perpendicular.

3. (5 Marks) Find all solutions, real and complex, of the equation $x^6 + 9x^4 - 4x^2 - 36 = 0$.

4. (9 Marks)

   (a) Sketch the graph of the parabola $y = -x^2 + 5x - 6$ indicating

       (i) vertex
       (ii) x-intercepts
       (iii) y-intercept
       (iv) axis.

   (b) Find the domain and range of the function $g(x) = \sqrt{-x^2 + 5x - 6}$

   (c) Find the domain of $h(x) = \ln(-x^2 + 5x - 6)$.

5. (5 Marks) A charter company charges $ 200 per person, and an additional $4 per person for each unsold seat. If the plane holds 100 passengers, find the number of unsold seats that produces the maximum revenue for the company.

6. (5 Marks)

   (a) Sketch the graph of $y = -x^3 + x^2 + 6x$.

   (b) Obtain the domain of $g(x) = \sqrt{-x^3 + x^2 + 6x}$.

7. (12 Marks) Solve for x. Round off calculator answers to 3 decimal places.

   (a) $\log(x^2 + 2x + 37) = 2$
   (b) $(x + 3)^{10} = 47352$
   (c) $4^{2x} = 7^{x+1}$
   (d) $\log_x 21 = \dfrac{-2}{3}$
   (e) $\log_{1.238} 7.365$

8. (5 Marks) The half-life of a radioactive substance is 11 months. How much of a 387 gram sample will remain at the end of 9 months. Give your answer to the nearest gram.

   Useful Information - radioactive decay formula $A(t) = A_0 e^{-kt}$.

9. (6 Marks) Find the rate of interest, to two decimals places, for money to double in 7 years if compounding is (a) quarterly, (b) continuously.

10. (7 Marks) *Do this problem **without** a calculator and indicate clearly how the answer was obtained.*

    The rate at which an infection spreads is proportional to both the number of people who have been infected and the number of people who have not yet been infected. Initially 1/10 of the population of a town is infected. After 2 weeks 1/4 of the population is infected. What fraction is infected after 4 weeks?

    Hint: $y(t) = \dfrac{N}{1 + Ce^{-kt}}$, where $y(t)$ is the number of people infected after t weeks, N is the total population of the town where the epidemic is confined, C and k are positive constants.

11. (7 Marks) Find the length of DC.
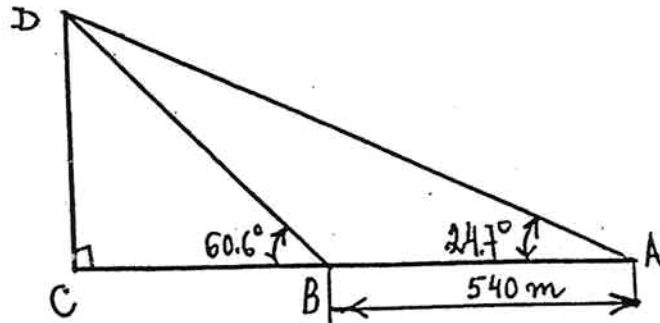
12. (8 Marks) The circular arc of a railroad curve has a chord AB of length 1000, and a central angle of 25°. Calculate

    (a) The radius of the arc.

    (b) The length of the circular arc AB.

    (c) The area of the sector ABC.
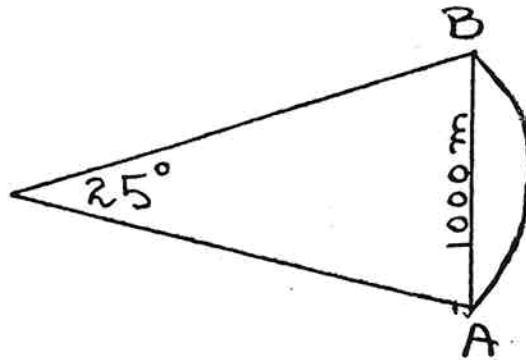
13. (4 Marks) Two planes leave Montreal with average speeds of 865 km/h. and 940 km/h. making an angle of 124° with each other. How far apart are the planes after 3 hours?

14. (6 Marks)

    (a) Let $y = -5\sin(6x + 3)$.
        (i) What is the amplitude?
        (ii) What is the period?
        (iii) What is the phase shift?

    (b) The displacement of an object is governed by damped harmonic motion. If the initial displacement is 5 cm, the damping constant is 1.4 and the frequency is $90Hz$ (cycles per second), obtain an expression for $y(t)$, the displacement after t seconds.

15. (8 Marks)

    (a) Verify the identity

$$\frac{1 + \tan^2\theta}{1 - \tan^2\theta} = \frac{1}{\cos^2\theta - \sin^2\theta}$$

    (b) Solve for x, giving all values for $0 \le x < 360°$ and $0 \le x < 2\pi$ radians

$$2\sin^2 x + 5\cos x - 4 = 0.$$

16. (8 Marks) Evaluate <u>without</u> using calculators

    (a) $\sin(2\cos^{-1}\frac{3}{5})$

    (b) $\tan(2\tan^{-1}\frac{4}{3})$

    (c) $\cos(\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{3}{5})$

Good Luck!