1. Write the following decimals as fractions in reduced form:

    (a) $0.\overline{36}$

    (b) $0.1\overline{36}$

2. Factor $(x^2 + 4xy + 4y^2) - (3x + 6y)$ completely over the integers.

3. Solve the system

$$3x - 7y = 13$$
$$4x + 3y = 5$$

4. Find the equations in slope-intercept form of each of the following lines:

    (a) the line through $(-1, 4)$ and $(-2, 1)$.

    (b) the line through $(4, -1)$ that is perpendicular to the line $y = -2x - 7$.

5. Determine the centre and radius of the circle $x^2 + y^2 + 4x - 6y - 3 = 0$.

6. Simplify completely, leaving your answer free of negative exponents or radicals:

$$\sqrt[6]{b} \cdot \left(\frac{\sqrt[3]{a^3 b^2}}{\sqrt{a^6 b^3}} \right)^{-1}$$

7. Determine the vertical and horizontal asymptotes of the rational function

$$y = \frac{2x}{x^2 - x - 6}$$

8. Solve $\dfrac{1}{x} + \dfrac{1}{x - 2} = \dfrac{4}{3}$.

9. Determine the natural domain of definition of each of the following functions. Explain your results!

    (a) $f(x) = \dfrac{\sqrt{x}}{2 - \sin x}$

    (b) $g(x) = \dfrac{1}{\log_3(x + 1)}$

10. Find all solutions of $\log_2(3x + 2) + \log_2(x + 1) = 2$.

11. Suppose that a certain radioactive substance decays exponentially according to the formula $y = 100 \cdot (2^{-0.04t})$, where t is in years. Compute its half-life. Your final answer should not contain any logarithms!

12. Suppose that t is an angle with $\pi < t < 2\pi$ and $\cos t = \frac{3}{5}$. Compute

   (a) $\tan t$
   (b) $\sin(2t)$

13. Prove the trigonometric identity $\sin^2 t = \dfrac{1 - \cos(2t)}{2}$.

14. Richard often flies from Center City to Clear Lake and returns on the same day. On a windless day, he can average 120 miles per hour for the round trip, while on a windy day, he averaged 140 miles per hour one way and 100 miles per hour the other. It took him a quarter of an hour longer on the windy day. How far apart are Center city and Clear Lake?

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-112A

FUNDAMENTALS OF MATHEMATICS

Examiner: Dr. A. Hundemer   Date: Wednesday, December 16, 1998
Associate Examiner: Professor N.G.F. Sancho   Time: 9:00 A.M. - 12:00 Noon.

INSTRUCTIONS

Calculators are not permitted
SHOW ALL YOUR WORK!

This exam comprises the cover and 2 pages of questions.